Math 127 **Name:**____________________________

Exam 4 - Version 1

06/10/2011

You MUST show appropriate work to receive credit

1. (5 points each) Solve the following systems of linear equations:

$$\begin{cases} 3x - 4y = 8 \\ 5x - 2y = 11 \end{cases}$$

We multiply the second equation by -2, giving the following:

$$\begin{cases} 3x - 4y = 8 \\ -10x + 4y = -22 \end{cases}$$

Using the addition method, this gives:

$-7x = -14$, or $x = 2$

Substituting this value into the first equation gives:

$3(2) - 4y = 8$, or $6 - 4y = 8$

Then $-4y = 2$, or $y = -\frac{2}{4} = -\frac{1}{2}$.

Hence the solution is: $\left(2, -\frac{1}{2}\right)$.

$$\begin{cases} -4x + 8y = 11 \\ 3x - 6y = 7 \end{cases}$$

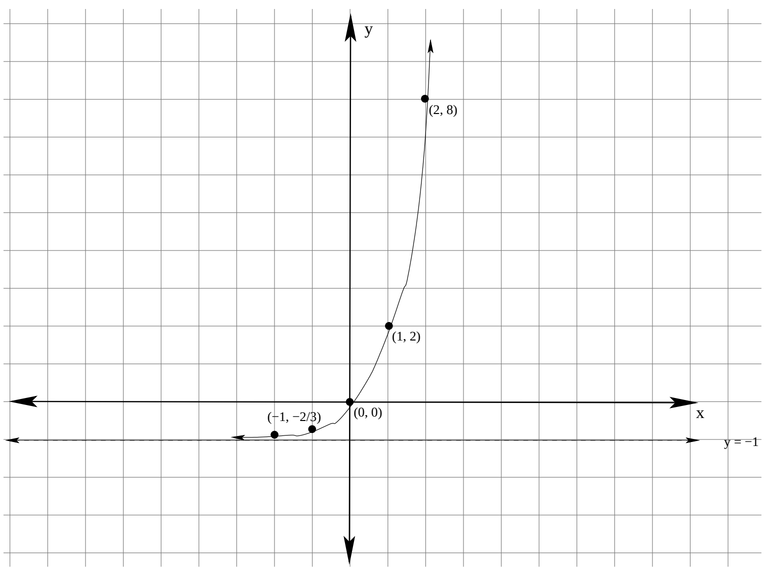
We again use the addition method. We multiply the first equation by 3 and the second by 4.

$$\begin{cases} -12x + 24y = 33 \\ 12x - 24y = 28 \end{cases}$$

Adding these gives $0 + 0 = 61$ which is false.

Hence this system has no solution.

2. (6 points) Graph the function $f(x) = 3^x - 1$. Graph and label the asymptote and at least 3 points on your graph. Also give the domain and range.



x	3^x	$f(x) = 3^x - 1$
-2	$3^{-2} = \frac{1}{9}$	$-\frac{8}{9}$
-1	$3^{-1} = \frac{1}{3}$	$-\frac{2}{3}$
0	$3^0 = 1$	0
1	$3^1 = 3$	2
2	$3^2 = 9$	8

Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Asymptote:
the horizontal line $y = -1$

3. (5 points) Suppose you have \$5,000 to invest. Find the amount you would have after 10 years if you deposit your \$5,000 in an account that pays 6% annual interest compounded monthly.

Recall that $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$. Here, $A = \$5000$, $r = 0,06$, $n = 12$, and $t = 10$.

Then we have $A = 5,000\left(1 + \frac{0.06}{12}\right)^{(12)(10)} \approx \$9,096.98$

4. (2 points each) Find the *exact value* of each of the following:

(a) $\log_2(16) = 4$

Since $2^4 = 16$

(b) $\log_{17}(1) = 0$

Since $17^0 = 1$

(c) $\log_\pi(\sqrt{\pi}) = \log_\pi\left(\pi^{\frac{1}{2}}\right) = \frac{1}{2}$

(d) $\log_3\left(\frac{1}{9}\right) = -2$

Since $3^{-2} = \frac{1}{9}$

(e) $\log_{16}(4) = \frac{1}{2}$

Since $16^{\frac{1}{2}} = 4$

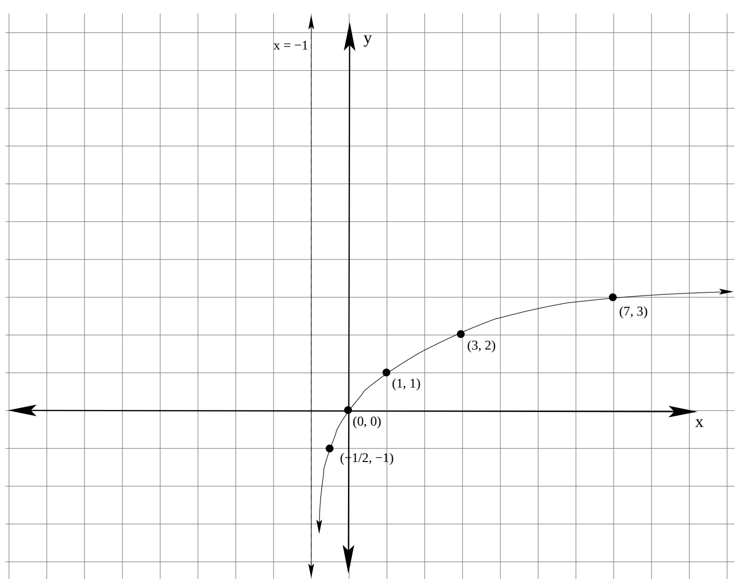
(f) $11^{\log_{11}(23)} = 23$

5. (5 points) Use properties of logarithms to expand the expression: $\log\left(\frac{\sqrt[3]{x^2+1}}{(4x-3y)^3}\right)$

$= \log\left(x^2+1\right)^{\frac{1}{3}} - \log(4x-3y)^3$

$= \frac{1}{3}\log\left(x^2+1\right) - 3\log(4x-3y)$

6. (6 points) In grid provided, sketch the graph of the function $g(x) = \log_2(x+1)$, clearly labeling at least 3 points. Also label any asymptotes, and give the domain and range.



$x+1$	x	$g(x) = \log_2(x+1)$
1	0	0
2	1	1
4	3	2
8	7	3
$\frac{1}{2}$	$-\frac{1}{2}$	-1
$\frac{1}{4}$	$-\frac{3}{4}$	-2

Domain: $(-1, \infty)$

Range: $(-\infty, \infty)$

Asymptote:
the vertical line $x = -1$

7. (5 points) Use the change of base formula to compute $\log_7(16)$ to 4 decimal places.

$\log_7(16) = \frac{\log 16}{\log 7} = \frac{\ln 16}{\ln 7} \approx 1.4248$

8. (6 points each) Solve the following equations (give exact answers whenever possible):

(a) $2^{3x-2} = 8^{1-2x}$

$2^{3x-2} = \left(2^3\right)^{1-2x}$

So $3x - 2 = 3(1 - 2x)$ or $3x - 2 = 3 - 6x$.

Then $9x = 5$, thus $x = \frac{5}{9}$

(b) $\log_4(3x - 5) = 2$

Rewriting in exponential form: $4^2 = 3x - 5$, so $16 = 3x - 5$

Then $21 = 3x$, thus $x = 7$.

Check: (we must check logarithmic equations)

$\log_4(3(7) - 5) = \log_4(16) = 2$ ✓

(c) $e^{5x} = 3^2$

$\ln\left(e^{5x}\right) = \ln\left(3^2\right)$

Then $5x = 2\ln 3$, so $x = \frac{2\ln 3}{5}$

(d) $5e^{3x-5} - 7 = 13$

$5e^{3x-5} = 20$, so $e^{3x-5} = \frac{20}{5} = 4$

$3x - 5 = \ln 4$, so $3x = 5 + \ln 4$

Thus $x = \frac{5+\ln 4}{3}$

(e) $\ln(x + 4) + \ln(x - 1) = \ln(x + 11)$

We begin by combining the left hand side into a single logarithmic expression:

$\ln\left((x + 4)(x - 1)\right) = \ln(x + 11)$

Then, using the 1-1 property, $(x + 4)(x - 1) = x + 11$

Expanding and combining terms, $x^2 + 3x - 4 = x + 11$

Hence $x^2 + 2x - 15 = 0$, or $(x + 5)(x - 3) = 0$

This given two potential solutions: $x = -5$ and $x = 3$.

Check: (we must check logarithmic equations)

Notice that $\ln(-5 + 4) = \ln(-1)$ is undefined, so we reject this solution.

$\ln(3 + 4) + \ln(3 - 1) = \ln(7) + \ln(2) = \ln(7 \cdot 2) = \ln 14$

$\ln(3 + 11) = \ln 14$ ✓

Thus there is one solution: $x = 3$.

(f) $3^{2x-1} = 5^{3x}$

Taking the logarithm of each side: $\ln 3^{2x-1} = \ln 5^{3x}$

Then $(2x - 1)\ln 3 = 3x \cdot \ln 5$, or, distributing this constant, $(2\ln 3)x - \ln 3 = (3\ln 5)x$

Moving terms, we have $(2\ln 3)x - (3\ln 5)x = \ln 3$

Therefore, $x(2\ln 3 - 3\ln 5) = \ln 3$

Thus $x = \dfrac{\ln 3}{2\ln 3 - 3\ln 5}$

9. Suppose that the population of Algebronia is growing according to the model $f(t) = 11e^{0.037t}$ where t is in years since 2000, and $f(t)$ is in millions of people.

   (a) (3 points) Find the population of Algebronia in the year 2000.

   Notice that the year 2000 corresponds to $t = 0$. so the population in 2000 is given by $f(0) = 11e^{0.037(0)} = 11e^0 = 11$, or 11 million people.

   (b) (3 points) Find the populations of Algebronia today (to the nearest person).

   Notice that today corresponds to $t = 2011 - 2000 = 11$. so the population today is given by $f(11) = 11e^{0.037(11)} \approx 16.52534516$
   or 16,525,345 people (notice that you were asked to find the population to the *nearest person*).

   (c) (5 points) Find the year that the population of Algebronia reaches 50 million people.

   To find the year in which the population will reach 50 million, we solve the equation $50 = 11e^{0.037t}$
   Then $\frac{50}{11} = e^{0.037t}$, or $\ln\left(\frac{50}{11}\right) = 0.037t$
   Hence $t = \dfrac{\ln\left(\frac{50}{11}\right)}{0.037} \approx 40.922$, or in the year 2041.

10. (7 points) Suppose that laboratory research has shown that a 7 gram sample of a newly discovered substance called Strawberronium is reduced down to 5 grams in 10 hours. Find the half life of Strawberronium.

    We will use the continuous decay model: $A = A_0e^{kt}$. To find the half life, we must first use the information we know about Strawberrionium to find the growth constant k. In the lab test, we had $A_0 = 7$ grams, $A = 5$ grams, and $t = 10$ hours.

    Then $5 = 7e^{10k}$, so $\frac{5}{7} = e^{10k}$, or $\ln\frac{5}{7} = 10k$. Thus $k = \frac{\ln\frac{5}{7}}{10} \approx -0.0336472$.

    Using this, we set up the half-life equation: $\frac{1}{2} \approx e^{-0.00336472t}$.

    Solving this for t, we have $\ln\frac{1}{2} \approx -0.00336472t$, so $t \approx \frac{\ln\frac{1}{2}}{-0.00336472} \approx 20.600$

    Hence the half-life of Strawberronium is about 20.6 hours.

Extra Credit: (5 points) Solve the Equation $\log_5(\log x) = 1$

First, re-writing in exponential form, $5^1 = \log x$, or $5 = \log x$
Rewriting again, we have $10^5 = x$, so $x = 100,000$.