Math 261
Exam 1 - Practice Problems

1. Given the points $A : (4, -2)$ and $B : (-2, 7)$:

   (a) Find an equation for the line containing A and B

   (b) Find the line that is perpendicular to the line you found in part (a) and containing the point $(1, -1)$

2. Find solutions to the inequality: $\dfrac{x^2 - 1}{x^2 + x - 6} \leq 0$.

3. Given the function $f(x) = \dfrac{1}{x - 2}$

   (a) What is the domain of f? Give your answer in interval notation.

   (b) Find $f(5)$ and $f(2a + 4)$

   (c) Find $\dfrac{f(a + h) - f(a)}{h}$ (be sure to simplify your answer).

4. Given that $f(x) = \dfrac{1}{2x - 3}$ and $g(x) = \sqrt{x^2 - 9}$

   (a) Find $f \circ g(2)$

   (b) Find the domain of $\dfrac{g}{f}$? Give your answer in interval notation.

5. Find the exact value of each of the following:

   (a) $\sin\left(\frac{7\pi}{4}\right)$

   (b) $\cos\left(\frac{4\pi}{3}\right)$

   (c) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

   (d) $\cos^{-1}(-1)$

   (e) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

6. Find all solutions to the following equations. Give the exact answers.

   (a) $2\sin 3x = \sqrt{3}$

   (b) $\sin^2(x) - \sin(x) = 0$

7. Find the values of x, y and z in the triangle shown below:
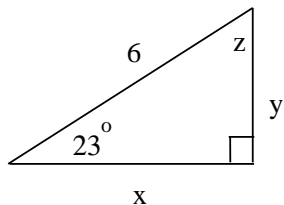
8. A function f is graphed below. Find the following:
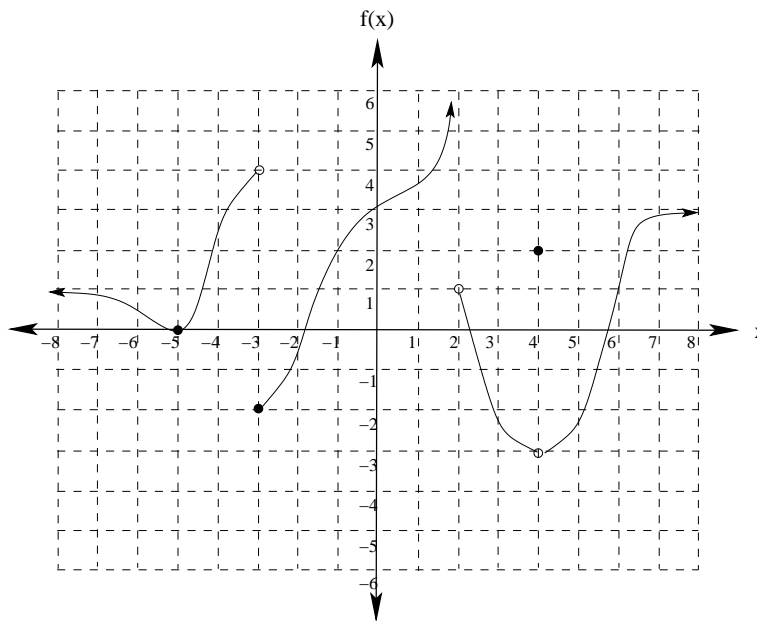
(a) $f(-5)$, $f(-3)$, and $f(4)$

(b) find the domain and range of f

(c) find the intervals where f is decreasing

(d) find $\lim\limits_{x \to 4} f(x)$

(e) find $\lim\limits_{x \to 2^-} f(x)$ and $\lim\limits_{x \to 2^+} f(x)$

(f) find $\lim\limits_{x \to -\infty} f(x)$ and $\lim\limits_{x \to \infty} f(x)$

(g) find the points where $f(x)$ is discontinuous, and classify each point of discontinuity.

9. Find the following limits:

(a) $\lim\limits_{x \to 2} \dfrac{3x + 7}{\sqrt{5x - 1}}$

(b) $\lim\limits_{x \to \frac{3}{2}} \dfrac{2x^2 + x - 6}{4x^2 - 4x - 3}$

(c) $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x^2 - x - 2}$

(d) $\lim\limits_{x \to -2^+} \sqrt{x + 2}$

(e) $\lim\limits_{x \to 3^+} \dfrac{4}{\sqrt{x - 3}}$

(f) $\lim\limits_{x \to \infty} \dfrac{(3x - 5)(2x - 3)}{(2x + 1)(3x - 2)}$

(g) $\lim\limits_{x \to \infty} \dfrac{(3x - 5)(2x - 3)}{(2x + 1)}$

10. Given the function

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 4 - x^2 & \text{if } x > 1 \end{cases}$$

(a) Graph $f(x)$.

(b) Find $\lim\limits_{x \to 1^-} f(x)$, $\lim\limits_{x \to 1^+} f(x)$, and $\lim\limits_{x \to 1} f(x)$

(c) Is $f(x)$ continuous at $x = 1$? Justify your answer.

11. Given that $f(x) = x^3 + 5$, $\lim\limits_{x \to 2} f(x) = 13$, and $\epsilon = .01$, find the largest δ such that if $0 < |x - 2| < \delta$, then $|f(x) - 13| < \epsilon$.

12. Use the formal definition of a limit to prove that $\lim\limits_{x \to 6} 5x - 21 = 9$.

13. Let $f(x) = \dfrac{x^2 - x - 2}{x^2 - 2x}$.

(a) Find the values of x at which f is discontinuous.

(b) Find all vertical and horizontal asymptotes of f.

14. Find the x values at which $f(x) = \dfrac{\sqrt{9 - x^2}}{x^4 - 16}$ is continuous.

15. Use the Intermediate Value Theorem to show $x^5 - 3x^4 - 2x^3 - x + 1 = 0$ has a solution between 0 and 1.