

1. Find the derivative  $y' = \frac{dy}{dx}$  for each of the following:

(a)  $y = \pi^2 x + \pi x^2$

(b)  $y = \cot x$

(c)  $y = \sqrt{x} \sec(x^2)$

(d)  $y = 2 \tan^3(2x^3)$

(e)  $y = \frac{x^2 - 7 \cos(3x)}{x + \sin(3 - 2x)}$

(f)  $x^2 y + 3xy - 5y^2 = 7$

(g)  $\cos^2(xy) = 1$

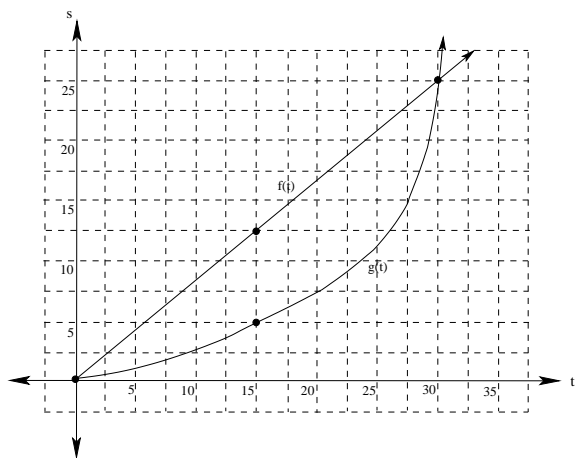
2. Use the formal limit definition of the derivative to find the derivative of the following:

(a)  $f(x) = x^2 - 3x$

(b)  $f(x) = \frac{2}{x - 3}$

(c)  $f(x) = \sqrt{x - 2}$

3. The position of two cars, car  $A$  and car  $B$ , both starting side by side on a straight road, is given by  $f(t)$  and  $g(t)$ , where  $f(t)$  is the distance traveled car  $A$  in feet, and  $g(t)$  is the distance traveled car  $B$  in feet, and  $t$  is in minutes (see the graph below):



(a) How fast is car  $A$  going at time  $t = 15$ ?

(b) Find the average rate of change of car  $B$  on the time interval  $[0, 15]$ .

(c) Which car is traveling faster at time  $t = 15$ ?

(d) Which car is traveling faster at time  $t = 30$ ?

(e) What can you say about the relative positions of the two cars at time  $t = 30$ ?

4. Use the quotient rule to derive the formula for the derivative of  $\tan(x)$ .
5. Use the product rule to prove that  $D_x[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$
6. If  $f(x) = \sqrt{3x-5}$ , find the intervals where  $f(x)$  is continuous, and find the intervals where  $f(x)$  is differentiable.
7. If  $f(x) = 3x^4 - 5x^2 + 7x - 12$ , use differentials to approximate  $f(1.1)$
8. Use differentials to approximate  $\sqrt{1.2}$ . How good is your approximation?
9. Use differentials to estimate  $\sqrt[3]{9}$ . How good is your approximation?
10. Suppose helium is being pumped into a spherical balloon at a rate of 4 cubic feet per minute. Find the rate at which the radius is changing when the radius is 2 feet.
11. Dr. Von Klausen has just invented a shrink ray and decides to try it out on a test object: a cylinder whose height is twice its radius. The shrink ray has been calibrated so that the proportions of the cylinder remain the same throughout the test. A few seconds into the test, the radius of the cylinder is decreasing at 2 inches per second, and the height is 4 inches. At what rate is the volume of the cylinder changing at that time? (Be sure to include units in your answer)
12. A company manufactures wooden cubes. Each side of the finished cubes are 5 inches long, with a maximum error of  $\pm 0.2$  inches per side. Use differentials to estimate the maximum error in the volume of the cube. Then, compare your estimate with the error in volume of a cube with largest possible volume manufactured within the given error tolerances.
13. Find the equation of the tangent line to the graph of  $f(x) = \tan(4x)$  when  $x = \frac{3\pi}{16}$
14. Find the equation of the tangent line to the graph of  $y = \sec(2x)$  when  $x = \frac{\pi}{6}$ .
15. Find the points on the graph of  $y = 2x^3 + 3x^2 - 72x + 5$  at which the tangent line is horizontal.
16. Find the equation of the tangent line to the graph of the relation  $x^2y + 3y^2 = 3x - 7$  at the point  $(2, -1)$
17. Draw the graph of a function  $f(x)$  that is continuous when  $x = 3$ , but is not differentiable when  $x = 3$ .
18. Find  $g'(2)$  if  $h(x) = f(g(x))$ ,  $f(3) = -2$ ,  $g(2) = 3$ ,  $f'(3) = 5$ , and  $h'(2) = -30$ .
19. Given that  $f(2) = -3$ ,  $g(2) = 2$ ,  $f'(2) = \frac{1}{2}$ ,  $g'(2) = -5$ , and  $h(x) = f(g(x))$ .  
Find the following:
 

(a) $(f - g)'(2)$	(b) $(fg)'(2)$
(c) $\left(\frac{f}{g}\right)'(2)$	(b) $h'(2)$
20. Find  $f^{(8)}(x)$  if  $f(x) = \sin(2x)$
21. Find  $f^{(13)}(x)$  if  $f(x) = x^{12} + 7x^5 - 3x^3 - 1$