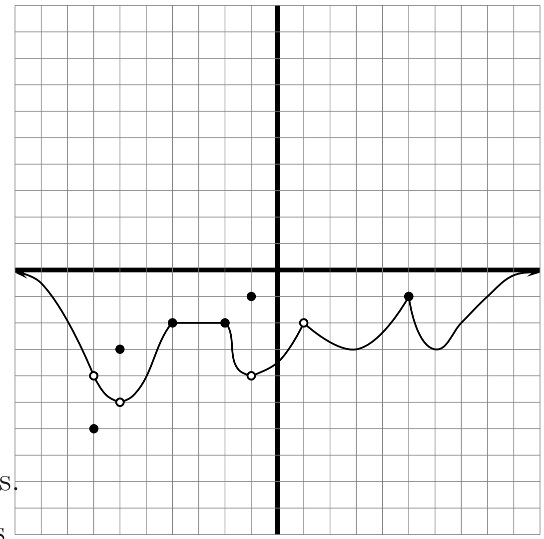
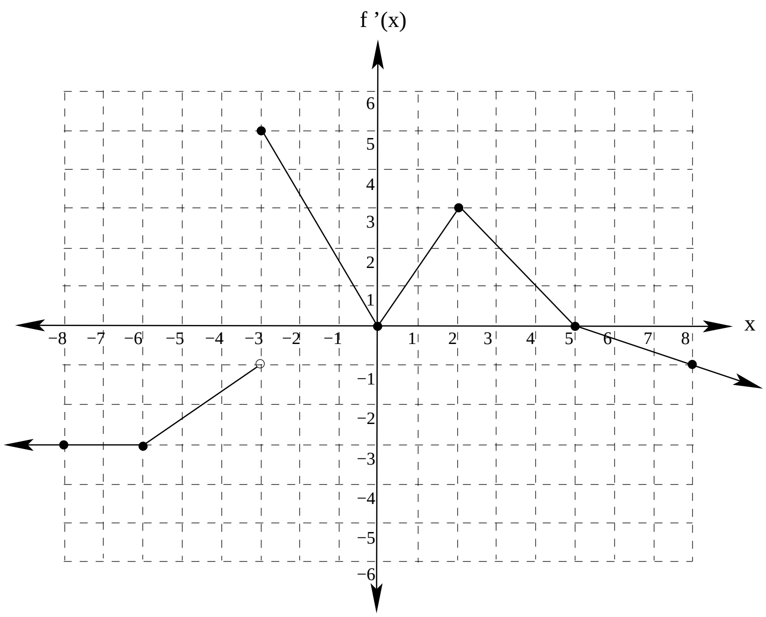
Math 261
Exam 3 - Practice Problems

1. The graph of f is given below. Answer the following questions.



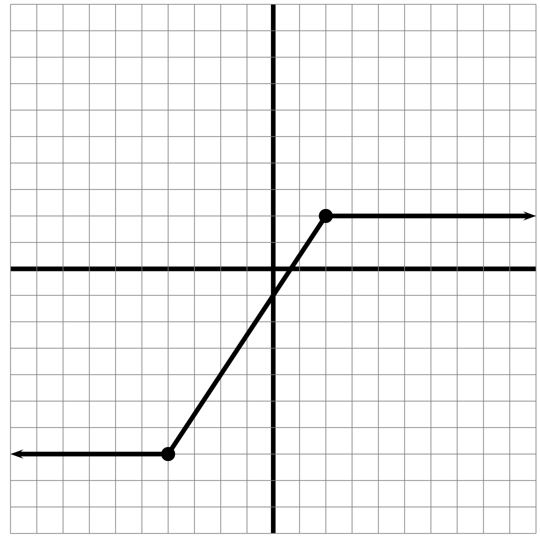
(a) Find the intervals where f is increasing.
(b) Find the intervals where f is decreasing.
(c) Find the intervals where f is constant.
(d) Find all local maximums.
(e) Find the location of all local maximums.
(f) Find all local minimums.
(g) Find the location of all local minimums.
(h) Find the absolute maximum and its location, if it exists.
(i) Find the absolute minimum and its location, if it exists.
(j) Find the absolute maximum on the interval $[3, 8]$, if it exists.
(k) Find the absolute minimum on the interval $[3, 8]$, if it exists.

2. Answer the questions below based on the graph of $f'(x)$ shown here:



(a) Find the intervals where $f(x)$ is decreasing.
(b) Give the x coordinates of the local extrema of f, and state whether each is a local maximum or a local minimum.
(c) Find the intervals where $f(x)$ is concave down.
(d) Give the x-coordinates of any inflection points.

3. The graph of f'' is shown. Answer the following questions.



(a) Sketch a possible graph of f'.

(b) Where is f concave up?

(c) Where is f concave down?

(d) Where is f' linear?

4. Find the absolute maximum and absolute minimum value of each of the following functions in the given interval:

(a) $f(x) = 2x^2 - 7x + 1$ on the interval $[0, 9]$

(b) $f(x) = x^3 - 9x + 5$ on the interval $[-2, 5]$

(c) $f(x) = \dfrac{x+3}{2x-1}$ on the interval $[1, 10]$

(d) $f(x) = \dfrac{x}{1+x^2}$ on the interval $[-2, 4]$

5. For each of the following, assuming that $f(x)$ is both continuous and differentiable everywhere, state what graph feature occurs at $f(2)$ (i.e. there is a local maximum, a local minimum, inflection point, or more information is needed):

(a) $f(2) = -5$, $f'(2) = 0$, and $f''(2) = -1$

(b) $f(2) = 7$, $f'(2) = -3$, and $f''(2) = 0$

(c) $f(2) = 1$, $f'(2) = 0$, and $f''(2) = 3$

6. Sketch the graph of a function $f(x)$ that satisfies the following:

Domain: $(-\infty, -2) \cup (-2, 4) \cup (4, \infty)$; x-intercepts: $(-4, 0), (-1, 0)$, and $(2, 0)$; y-intercept: $(0, 4)$

Increasing on: $(-2, 0)$; Decreasing on: $(-\infty, -2) \cup (0, 4) \cup (4, \infty)$

Concave up on: $(-1, 0) \cup (0, 2) \cup (4, \infty)$; Concave down on: $(-\infty, -2) \cup (-2, -1) \cup (2, 4)$

Local Max: $(0, 4)$, Local Mins: none

$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = 2$; $\lim_{x\to-2^-} f(x) = -\infty$; $\lim_{x\to-2^+} f(x) = -\infty$; $\lim_{x\to4^-} f(x) = -\infty$; $\lim_{x\to4^+} f(x) = \infty$

7. Find any local extrema of the following functions. Classify each as a maximum or a minimum.

(a) $f(x) = x^3 - 7x^2 - 5x + 6$

(b) $f(x) = x^3\sqrt{x} - 14x^2 + 10$

8. A particle is moving along a straight line during a sixty second time interval. Every ten seconds the position, velocity, and acceleration were measured and recorded in a table below. Answer each question below and give a justification for your answers.

t (*seconds*)	0	10	20	30	40	50	60
$s(t)$ (*feet*)	0	5	25	40	90	75	25
$v(t)$ (*feet/sec*)	0	2	3	2	1	-1	1
$a(t)$ (*feet/sec*^{2})	10	5	3	1	1	2	5

(a) Is there a time when $s(t) = 20$?

(b) Is there a time when $v(t) = 5$?

(c) Is there a time when $a(t) = 0$?

9. Find a number c in the given interval that satisfies the Mean Value Theorem for the function and interval given, or explain why the Mean Value Theorem does not apply.

(a) $f(x) = x^3 - x$ on $[-1, 1]$

(b) $f(x) = \dfrac{x+1}{x-1}$ on $[2, 5]$

(c) $f(x) = \dfrac{x+1}{x-1}$ on $[-1, 2]$

(d) $h(\theta) = \sin\theta + \cos\theta$ on $[0, 2\pi]$

10. For each of the following functions:

- Find the intercepts of $f(x)$.
- Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.
- Find and classify all local extrema of $f(x)$.
- Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.
- Find any inflection points of $f(x)$.
- Sketch the graph of $f(x)$ accurately enough to show all relative extrema and inflection points.

(a) $f(x) = 3x^2 - 6x - 5$

(b) $f(x) = \dfrac{x+2}{x-1}$

(c) $f(x) = x^4 - 24x^2$.

(d) $f(x) = x^4 - 4x + 2$

11. Solve the following Optimization problems:

   (a) A box with square base and open top is to have a volume of 4 cubic feet. Find the dimensions of the box that will minimize the surface area of the box.

   (b) A field that is to be used for a play area for a large dog is to be fenced in. The field is adjacent to a house, and the side with the house will need no fence. If 500 feet of fence is available, find the dimensions of the field that maximum the area in which the dog can play in.

   (c) Suppose a farmer has an apple orchard that has trees planted at a density of 30 trees per acre. The orchard currently yields 12 bushels per tree. For each additional tree planted (per acre), the average yield per tree will be reduced by 0.1 bushels. Find the number of additional trees (per acre) that should be planted in order to maximize the number of apples (per acre) produced by this orchard. Also find the yield per acre after the new trees have been planted.

   (d) Postal regulations require a parcel sent through US mail to have a combined length and girth of at most 108 inches. Find the dimensions of the cylindrical package of greatest volume that may be sent through the mail.

   (e) The owner of a yacht charges \$600 per person if exactly 20 people sign up for a cruise. If more than 20 sign up for the cruise (up to a maximum capacity of 90), then the fare for each passenger is discounted by \$4 for each additional passenger beyond 20. Assuming that at least 20 people sign up, determine the number or passengers that will maximize the revenue for the cruise and find the fare for each passenger.