Antiderivatives and Integration

A. Antiderivatives

A function F is an **antiderivative** of f if $F'(x) = f(x)$.

Example: $F(x) = x^3$ is an antiderivative of $f(x) = 3x^2$, since $F'(x) = 3x^2 = f(x)$.

Note: If $F(x)$ is an antiderivative of f, then so is $F(x) + C$ for any constant C.

B. Indefinite Integrals

The notation $\displaystyle\int f(x) \, dx = F(x) + C$ is used to denote the indefinite integral of the function $f(x)$ with respect to the variable x. Here, $F(x) + C$ is the general antiderivative of $f(x)$.

C. Integration Rules

1. Integrating a constant: $\displaystyle\int a \, dx = ax + C$ for any constant a.

   Example: $\displaystyle\int \frac{3}{2} \, dx = \frac{3}{2}x + C$

2. Integrating Power Functions: $\displaystyle\int x^r \, dx = \frac{1}{r+1} x^{r+1} + C$

   Example: $\displaystyle\int x^{\frac{3}{2}} \, dx = \frac{2}{5} x^{\frac{5}{2}} + C$

3. Constant Multiples: $\displaystyle\int a f(x) \, dx = a \int f(x) \, dx$ for any constant a.

   Example: $\displaystyle\int 7 x^{\frac{3}{2}} \, dx = 7 \int x^{\frac{3}{2}} \, dx = (7)\frac{2}{5} x^{\frac{5}{2}} + C = \frac{14}{5} x^{\frac{5}{2}} + C$

4. Sums and Differences: $\displaystyle\int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx$,

   and $\displaystyle\int f(x) - g(x) \, dx = \int f(x) \, dx - \int g(x) \, dx$ for any functions f and g.

   Example: $\displaystyle\int 3x^2 + x^{\frac{3}{2}} \, dx = \int 3x^2 \, dx + \int x^{\frac{3}{2}} \, dx = x^3 + \frac{2}{5} x^{\frac{5}{2}} + C$