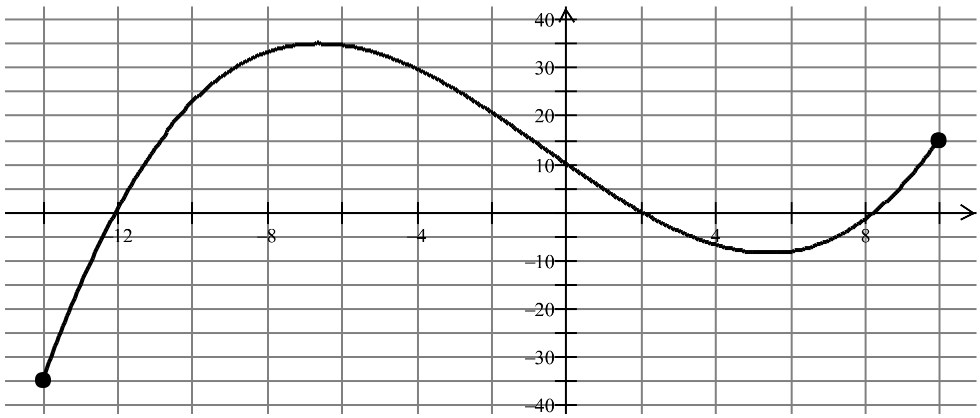
Math 261 – Summer Lab 12 **Mean Value Theorem** Name______________________

Show all work for credit.

1. Consider the following graph of a function f:
   (a) Make a sketch illustrating the conclusion of the Mean Value Theorem.

   (b) Determine the average rate of change.

   (c) Estimate all values c that satisfy the conclusion of the MVT.



2. Find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   (a) $s(t) = 3t^2 - 2t + 15$ on [0, 5] (b) $a(\varphi) = \sin \varphi$ on $\left[0, \frac{3\pi}{2}\right]$

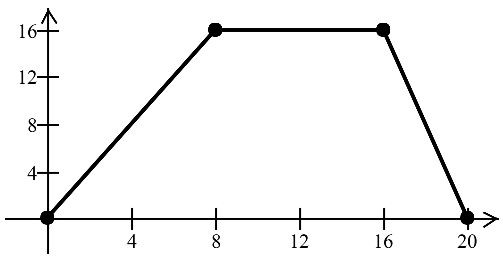
3. (From the 2007 AP Calculus AB exam.) Assume that the functions f and g are differentiable for all real numbers, and that g is strictly increasing. The table below gives values of the functions and their first derivatives at selected values of x. The function h is defined by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value k for $1 < k < 3$ such that $h'(k) = -5$.

4. The graph below models the velocity (in meters per second) from 0 to 20 seconds of a car on a straight road between two stop signs. Label all solutions with the appropriate labels.



(a) Find the acceleration at each of the following times. If the acceleration does not exist, explain why.

(i) 8 seconds

(ii) 18 seconds

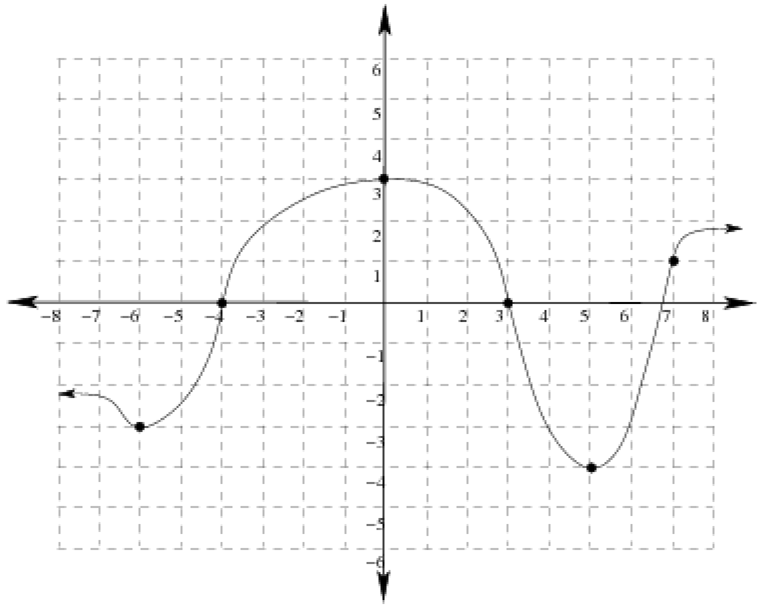
(b) Write a piece-wise defined function for the acceleration.

(c) Find the average rate of change of the velocity over the time interval from 10 seconds to 18 seconds.

(d) Does the Mean Value Theorem guarantee a value c, for $8 < c < 20$? Either find all values c or explain why there are none.

5. Answer the following questions based on the graph of f' (the derivative of function f) shown below:

(a) Find the intervals on which f is increasing.



(b) Find the intervals on which f is decreasing.

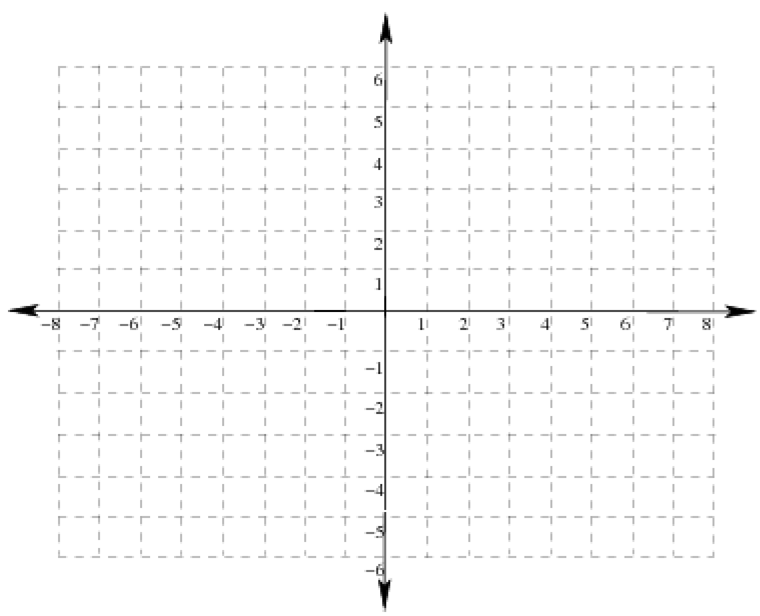
(c) Identify the location of a local maxima for f.

(d) Identify the location of a local minima for f.

(h) Find the intervals on which $f''(x) < 0$.

(e) When is f increasing the fastest?

(f) When is f decreasing the fastest?



(g) On the given coordinate plane, sketch a possible graph for f given that $f(0) = 2$.

6. For $s(t) = \dfrac{t^2}{t-3}$, (i) find all critical numbers, (ii) determine where the function is increasing and where it is decreasing, (iii) determine whether each critical number represents a local maximimum, local minimum, or neither, and (iv) use the information to sketch the graph of the function.

7. Sketch a graph of a function g that satisfies all of the following properties:
$|g(x)| < 2$ for all x; $g(-3) = g(-1) = 0$; $g'(x) < 0$ for $x < -2$ and $g'(x) > 0$ for $x > -2$;
$g(-2)$ is undefined; and $\lim_{x \to -2^-} g(x) > \lim_{x \to -2^+} g(x)$

8. A section of rollercoaster is in the shape of $y = -\frac{3}{5}x^5 + 5x^3 - 12x + 70$, where $-3 < x < \frac{5}{2}$. Find all local extrema. Where are the highest and lowest points on this section of the rollercoaster? Sketch a graph of this section of the rollercoaster. Where would you expect the rollercoaster to be gaining the most speed?