Show all work for credit.

Basic steps to solve optimization problems:
   (a) Read the problem and organize the key facts and quantities described.
   (b) Introduce variables for unknown quantities.
   (c) Draw and label a diagram.
   (d) Express all known facts and any relationships between the variables introduced.
   (e) Determine which unknown is to be optimized, then express it as a function.
   (f) Use the first or second derivative test, and test endpoints to find the correct extreme value.
   (g) Make sure you clearly justify the solution.
   (h) State the solution in a complete sentence.

1. Find the real number, which, when added to its square, gives the smallest sum.

2. A real estate company owns 180 apartments that are fully occupied when the rent is $300 per month. The company estimates that for each $10 increase in rent, five apartments will become unoccupied. What rent should be charged in order to obtain the largest gross income?

3. A wire 60 inches long is to be cut into two pieces. One piece will be bent into the shape of a circle and the other into an equilateral triangle. Where should the wire be cut to minimize the total area of the circle and the triangle? Where should the wire be cut to maximize the total area?

4. A concessionaire had been selling 5000 hotdogs at each football game at $2 apiece. When the concessionaire raised the price to $2.80 apiece, sales dropped to only 4000 per night. Assume a linear relationship between the price and sales. If the fixed costs each night are $4000, and each hotdog costs the concessionaire $1, what price per hotdog will maximize the nightly profit?

5. A gutter is to be made from a long strip of tin that is eighteen inches wide by bending up the sides so that its cross section is an isosceles trapezoid with the bottom and two sides all the same length. At what angle should the sides be bent to maximize the volume of the gutter?

6. An offshore oil well is located in the ocean 5 miles from shore. The oil is to be pumped to a point 8 miles up the coast to a storage facility (assume the coastline is straight). The cost of laying pipe is $1,000,000 per mile underwater and $750,000 per mile over land. Where should the pipe reach the shoreline? What is the cost of laying the pipeline?