Math 261 - Summer Lab 15 **Rectilinear Motion and Newton's Method** Name: _____

Show all work for credit. Also, give exact answers unless otherwise noted.

1. The height of a projectile fired from level ground in time t, at an initial angle θ from the horizontal, and at an initial velocity v_0 is often approximated by:

$$y(t) = v_0 t \sin(\theta) - \frac{1}{2} g t^2$$

where g is the acceleration due to gravity. The horizontal distance the projectile travels from the point at which it is fired is approximate by:

$$x(t) = v_0 t \cos(\theta)$$

.

(a) Find the projectile's vertical velocity as a function of time.

(b) Find the projectile's vertical acceleration as a function of time.

(c) Find the projectile's horizontal velocity as a function of time.

(d) Find the projectile's horizontal acceleration as a function of time.

(e) Find the projectile's speed as a function of time.

(f) Find the projectile's absolute acceleration as a function of time.

(g) When does the projectile attain its maximum height?

(h) Find the projectile's maximum height.

(i) When does the projectile hit the ground?

(j) Find the range of the projectile.

(k) Assume a projectile is fired at an angle of 35° with a velocity of 900 meters per second. Use your formulae to find the maximum height and range of the projectile for each of the following bodies from our solar system. Round all solutions to three significant digits.

i. Earth with $g = 9.8\frac{\text{m}}{\text{s}^2}$

ii. Earth's moon with $g = 1.62\frac{\text{m}}{\text{s}^2}$

iii. Neptune with $g = 11.4\frac{\text{m}}{\text{s}^2}$

iv. Saturn's moon Mimas with $g = 0.0651\frac{\text{m}}{\text{s}^2}$ (Mimas has a gigantic crater than covers about a quarter of its surface.)

v. The radius of Mimas is approximately $196,000$ meters. Compare this to your previous answer and give a reasonable explanation.

2. Approximate $\sqrt{21}$ using Newton's method. Give your answer to six decimal places. Show your work, including your initial guess and all intermediate results. [Hint: What function $f(x)$ could you use to carry out this approximation?]