Math 261
Properties of Limits

Theorem 2.7 $\lim_{x\to a} c = c$, and $\lim_{x\to a} x = a$.

Theorem 2.8 If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist, then:

- $\lim_{x\to a} f(x) + g(x) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.
- $\lim_{x\to a} f(x) \cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$.
- $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided $\lim_{x\to a} g(x) \neq 0$.
- $\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x)$ for any constant c.
- $\lim_{x\to a} f(x) - g(x) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$.

Theorem 2.9 If m, b, and a are real numbers, then $\lim_{x\to a} mx + b = ma + b$.

Theorem 2.10 If n is a positive integer then:

- $\lim_{x\to a} x^n = a^n$.
- $\lim_{x\to a} (f(x))^n = \left(\lim_{x\to a} f(x)\right)^n$, provided $\lim_{x\to a} f(x)$ exists.

Theorem 2.11 If $f(x)$ is a polynomial, then $\lim_{x\to a} f(x) = f(a)$.

Corollary 2.12 If $q(x) = \frac{f(x)}{g(x)}$ is a rational function, then $\lim_{x\to a} q(x) = q(a) = \dfrac{f(a)}{g(a)}$, provided $g(a) \neq 0$.

Theorem 2.13 If $a > 0$ and n is a positive integer or if $a \leq 0$ and n is an odd positive integer, then $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$.

Theorem 2.14 If $\lim_{x\to a} f(x)$ exists then $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ provided either n is an odd positive integer or n is an even positive integer and $\lim_{x\to a} f(x) > 0$.

Theorem 2.15: The Sandwich Theorem Suppose $f(x) \leq h(x) \leq g(x)$ for every x in an open interval containing a, except possibly at a itself. If $\lim_{x\to a} f(x)L = \lim_{x\to a} g(x)$, then $\lim_{x\to a} h(x) = L$.

Examples: