Math 127 - College Algebra
Handout: Algebraic Expressions, Sets, Mathematical Models, and Real Numbers

• An **algebraic expression** is a combination of variables and numbers using the operations of addition, subtraction, multiplication or division, as well as powers or roots.

Examples: $3x$, $2x - y$, $x^2 - 4xy$, $\sqrt{2x} + 7$

• **Evaluating an algebraic expression** is finding the value of the expression for given values for each of the variables represented in the expression.

Note: Use "Order of Operations" [PEMDAS] when evaluating any algebraic expression.

1. Perform operations within grouping symbols like parentheses, brackets, and braces beginning within the innermost grouping and working outwards.

2. Evaluate all exponential expression before other operations.

3. Perform multiplications and divisions, working from left to right.

4. Perform additions and subtractions, working from left to right.

Examples:

1. Evaluate $\sqrt{2x} + 7$ when $x = 8$

2. Evaluate $x^2 - 4xy$ when $x = 3$ and $y = -2$

3. Evaluate $2x^2 - 7(x - 3)^3$ when $x = 2$

• A **set** is a collection of objects of some type. We usually use capital letters to denote sets. Individual elements are called **elements**.

Examples: $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$, $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$, $\{x \mid x^2 < 16 \text{ and } x \text{ is an integer }\}$

Notation:

• $A \cap B$ is the **intersection** of the sets A and B. That is, all of the elements that the two sets have in common.
• $A \cup B$ is the **union** of the sets A and B. That is, all of the elements that are in either of the two sets.
• $\emptyset$ denotes the empty set, a set that has *no* elements in it.

Examples:

1. $\{2, 4, 7, 9\} \cup \{1, 4, 5, 7\} = \{1, 2, 4, 5, 7, 9\}$

2. $\{2, 4, 7, 9\} \cap \{1, 4, 5, 7\} = \{4, 7\}$

3. $\{2, 3, 5, 7\} \cap \{1, 6, 8, 9\} = \emptyset$

• Recall the definitions of Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, and Real Numbers [See page 7 in your textbook]

Given the set: $\{-3, 0, \frac{1}{2}, \frac{16}{2}, \sqrt{2}, -\frac{7}{2}, .314159, \sqrt{16}, \sqrt{-1}, \pi\}$:

1. Which elements of this set are integers?

2. Which elements of this set are rational?

3. Which elements of this set are irrational?

• The symbols: $<, \leq, >, \geq$ arise from the standard ordering of the real line and allow us to indicate the relative size of any pair of real numbers.

• **Absolute Value:**

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Examples: (a) $|7| = 7$ (b) $|-4| = 4$

Properties:

1. $|a| \geq 0$

2. $|-a| = |a|$

3. $|ab| = |a||b|$

4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

5. $|a + b| \leq |a| + |b|$

Evaluating Absolute Value Expressions:

$(i) |\pi - 3| = \pi - 3$ $(ii) |3 - \pi| = \pi - 3$

$(iii) \dfrac{|-4| - |7|}{|-4 - 7|} = ?$

• Review the Properties of Real Numbers on pages 12-13 in your textbook.

• Using the Properties of Real Numbers and Order of Operations, we can simplify many algebraic expressions.

Example: Simplify the algebraic expression $7x^2 - 3 + [3(x^2 - 1) - 10]$