

## Math 127 - College Algebra

### Handout: Factoring

#### Special Factoring Formulas

Difference of Squares:  $u^2 - v^2 = (u + v)(u - v)$

**Example:**  $36x^2 - y^2 = (6x + y)(6x - y)$

Perfect Square:  $u^2 + 2uv + v^2 = (u + v)^2$

**Example:**  $4x^2 + 12x + 9 = (2x + 3)^2$

Sum of Cubes:  $u^3 + v^3 = (u + v)(u^2 - uv + v^2)$

**Example:**  $8x^3 + y^3 = (2x + y)(4x^2 - 2xy + y^2)$

Difference of Cubes:  $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$

**Example:**  $27x^3 - 8y^3 = (3x - 2y)(9x^2 + 6xy + 4y^2)$

(Non Rule) Sum of Squares:  $u^2 + v^2$  Does Not Factor.

#### Factoring Methods:

##### 1. Greatest Common Factors

In this factoring method, you look at the expression to be factored and factor out the greatest common factor shared by *all* the terms in the expression (if there is one other than 1). This method should be employed before any other factoring method.

**Example:** Given the expression  $15x^4y - 6x^3y^2 + 21x^2y^3$ , the greatest common factor is:  $3x^2y$

The result of factoring out this common factor is:  $3x^2y(5x^2 - 2xy + 7y^2)$

##### 2. Factoring by Grouping

This factoring method is used in expressions with an *even number of terms* (four or more).

To use this technique, follow these steps:

- Group the terms together in two equal halves. Include in each half the terms that seem to have the most in common with each other.
- Find and factor out the greatest common factor in each half separately.
- After factoring out the greatest common factor in each half, look to see if the remaining grouped terms are the same [if they differ only by a minus sign, factor that out of one half].
- Complete this method by grouping together the greatest common factors into a binomial term.

**Example:** Given the expression  $x^3 - 4xy + 5x^2 - 20y$ :

- Group the terms together in two equal halves:  $x^3 + 5x^2$  and  $-4xy - 20y$
- Find and factor out the greatest common factor in each half separately:  $x^2(x + 5)$  and  $-4y(x + 5)$
- Look to see if the remaining grouped terms are the same: both have the term  $(x + 5)$
- Complete this method by grouping together the greatest common factors into a binomial term:  $(x^2 - 4y)(x + 5)$

### 3. Factoring Trinomials (the “ $ac$ split”)

This factoring method is used to factor quadratic expressions of the form:  $ax^2 + bx + c$  or  $au^2 + buv + cv^2$ .

Follow these steps:

- (a) Write down all of the possible ways of factoring the product  $ac$
- (b) Look for a combination that adds up to the  $b$  term
- (c) Split the  $b$  term into the sum of two terms that you found.
- (d) Complete this method by using factoring by grouping.

**Example:** Given the expression  $6x^2 - 7x - 20$ :

1. Write down all of the possible ways of factoring  $ac$

$$ac: 6 \cdot (-20) = -120.$$

This can be factored as:  $1 \cdot (-120)$ ,  $-1 \cdot 120$ ,  $2 \cdot -60$ ,  $-2 \cdot 60$ ,  $3 \cdot -40$ ,  $-3 \cdot 40$

$4 \cdot -30$ ,  $-4 \cdot 30$ ,  $6 \cdot -20$ ,  $-6 \cdot 20$ ,  $8 \cdot -15$ ,  $-8 \cdot 15$ ,  $10 \cdot -12$ ,  $-10 \cdot 12$

2. Look for a combination that adds up to the  $b$  term

We first check:  $1 - 120 = -119 \neq -7$  or  $-1 + 120 = 119 \neq -7$

Then we check  $2 - 60 = -58 \neq -7$  or  $-2 + 60 = 58 \neq -7$

Continuing in this way, we eventually get to:

$$8 - 15 = -7$$

3. Split the  $b$  term into the sum of two terms that you found.

Since the combination we found was 8 and  $-15$ , we write  $6x^2 + 8x - 15x - 20$

4. Complete this method by using factoring by grouping.

Grouping, we get  $6x^2 + 8x - 15x - 20$

$$= 2x(3x + 4) - 5(3x + 4)$$

Then the factorization is:  $(3x + 4)(2x - 5)$