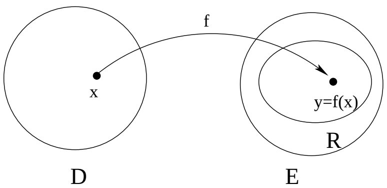
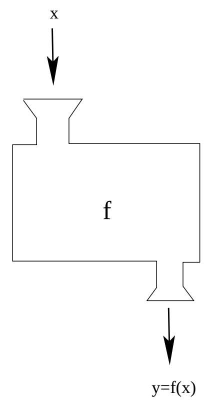
Math 127
Functions

Definition: A function f from a domain set D to a set E is a correspondence that assigns to each element x of D exactly one element y of E. We call x the **argument** of f and y the **value** of f at x. The **range** of f is the subset R of E consisting of all y values that corresponding to an x in the domain D.





• To evaluate a function, we input an x-value and find the corresponding value by applying the "rule" for the function to that input.

• Sometimes we also want to work backwards, that is, given an **output**, we try to find the *input(s)* that lead to that particular output.
• To find the domain of a function, we carefully analyze the function "rule" and find any x values that do not have corresponding outputs. Two things we look for in particular are *division by zero* and *even roots of negative numbers*.
Example 1:
Suppose $f(x) = \frac{x+1}{x-1}$. Then:

$f(2) = \frac{2+1}{2-1} = \frac{3}{1} = 3$

$f(-1) = \frac{-1+1}{-1-1} = \frac{0}{-2} = 0$

$f(2a-1) = \frac{2a-1+1}{2a-1-1} = \frac{2a}{2a-2} = \frac{a}{a-1}$

If $f(x) = 2$, that what is x?

$\frac{x+1}{x-1} = 2$, so $x + 1 = 2(x - 1) = 2x - 2$.

Then $x + 3 = 2x$, or $3 = x$. Check: $f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$.

The domain of f is ? ________

Example 2:
Let $g(x) = \frac{\sqrt{3x-3}}{x^2+2x-3}$ Find:

• $g(4)$

• $g(1)$

• the domain of $g(x)$

Alternate Definition of a Function: A *function* with domain D is a set W of ordered pairs such that, for each x in D, there is exactly one ordered pair (x, y) in W having x in the first coordinate.

Note: A *linear function* is any function of the form $f(x) = ax + b$.

II. Graphs of Functions

Definition:

The **graph** of a function is the set of all points $(x, f(x))$ (where x is in the domain D of f).

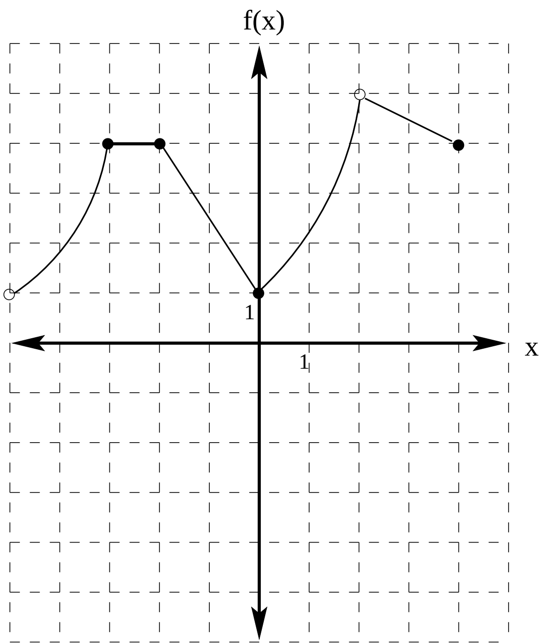
The Vertical Line Test:

A graph of points in the plane is the graph of a function if and only if every vertical line intersects the graph *at most* once.

Definitions:

A function is **increasing** on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
A function is **decreasing** on an interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.
A function is **constant** on an interval I if $f(x_1) = f(x_2)$ for all x_1, x_2 in I.

Example:



Find:
(a) $f(4)$

(b) x if $f(x) = 4$

(c) the domain of f

(d) the range of f

(e) the intervals where $f(x)$ is increasing