

D. Absolute Value

Definition:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Examples: (a) $|7| = 7$

(b) $|-4| = 4$

Properties:

1. $|-a| = |a|$
2. $|ab| = |a||b|$
3. $|\frac{a}{b}| = \frac{|a|}{|b|}$
4. $|a + b| \leq |a| + |b|$

Evaluating Absolute Value Expressions:

(i) $|\pi - 3| = \pi - 3$ (ii) $|3 - \pi| = \pi - 3$

$$(iii) \frac{|-4| - |7|}{|-4 - 7|} = \frac{4 - 7}{11} = \frac{-3}{11}$$

Absolute Value Equations and Inequalities:

Note: Since absolute value acts differently on positive numbers than it does on negative numbers, we will need to look at positive and negative cases in order to solve equations and inequalities involving absolute value.

Examples:

1. $|x + 3| = 4$

positive case:

$$\begin{aligned} x + 3 &= 4 \\ -3 & \quad -3 \\ x &= 1 \end{aligned}$$

negative case:

$$\begin{aligned} -(x + 3) &= 4 \\ \text{or} \\ x + 3 &= -4 \\ -3 & \quad -3 \\ x &= -7 \end{aligned}$$

2. $|2x - 3| \leq 7$

positive case:

$$\begin{aligned} 2x - 3 &\leq 7 \\ +3 & \quad +3 \\ 2x &\leq 10 \\ \text{so } x &\leq 5 \end{aligned}$$

negative case:

$$\begin{aligned} -(2x - 3) &\leq 7 \\ \text{or} \\ 2x - 3 &\geq -7 \\ +3 & \quad +3 \\ 2x &\geq -4 \\ \text{so } x &\geq -2 \end{aligned}$$

Thus $-2 \leq x \leq 5$

3. $|3x + 2| < -4$

Solution: ??