Math 127
Long Division or Polynomials and Synthetic Division Handout

Long Division Examples:

1. Find: $\dfrac{5x^3-4x^2+7x-13}{x+2}$

$$\begin{array}{r}
5x^2-14x+35 \\
x+2\,\big)\overline{\,5x^3-4x^2+7x-13} \\
\underline{-5x^3-10x^2} \\
-14x^2+7x \\
\underline{14x^2+28x} \\
35x-13 \\
\underline{-35x-70} \\
-83
\end{array}$$

$= 5x^2-14x+35+\frac{-83}{x+2}$

2. Find: $\dfrac{2x^4-x^2+7x-3}{x-3}$

$$\begin{array}{r}
2x^3+6x^2+17x+58 \\
x-3\,\big)\overline{\,2x^4-x^2+7x-3} \\
\underline{-2x^4+6x^3} \\
6x^3-x^2 \\
\underline{-6x^3+18x^2} \\
17x^2+7x \\
\underline{-17x^2+51x} \\
58x-3 \\
\underline{-58x+174} \\
171
\end{array}$$

$= 2x^3+6x^2+17x+58+\frac{171}{x-3}$

3. Find: $\dfrac{4x^5-2x^4+x^2-11}{2x+3}$

$$\begin{array}{r}
2x^4-4x^3+6x^2-\tfrac{17}{2}x+\tfrac{51}{4} \\
2x+3\,\big)\overline{\,4x^5-2x^4+x^2\phantom{-\tfrac{17}{2}x}-11} \\
\underline{-4x^5-6x^4} \\
-8x^4 \\
\underline{8x^4+12x^3} \\
12x^3+x^2 \\
\underline{-12x^3-18x^2} \\
-17x^2 \\
\underline{17x^2+\tfrac{51}{2}x} \\
\tfrac{51}{2}x-11 \\
\underline{-\tfrac{51}{2}x-\tfrac{153}{4}} \\
-\tfrac{197}{4}
\end{array}$$

$= 2x^4-4x^3-6x^2-\frac{17}{2}x+\frac{51}{4}+\frac{-197}{4(2x+3)}$

4. Find: $\dfrac{3x^4-5x^3+7x^2-5}{x^2+2x-1}$

$$\begin{array}{r}
3x^2-11x+32 \\
x^2+2x-1\,\big)\overline{\,3x^4-5x^3+7x^2-5} \\
\underline{-3x^4-6x^3+3x^2} \\
-11x^3+10x^2 \\
\underline{11x^3+22x^2-11x} \\
32x^2-11x-5 \\
\underline{-32x^2-64x+32} \\
-75x+27
\end{array}$$

$= 3x^2-11x+32+\frac{-75x+27}{x^2+2x-1}$

Synthetic Division Examples:

1. Find: $\dfrac{4x^2+3x-5}{x-2}$

	4	3	−5
2		8	22
	4	11	17

$= 4x+11+\frac{17}{x-2}$

2. Find: $\dfrac{x^3-3x^2+4x-1}{x+3}$

	1	−3	4	−1
−3		−3	18	−66
	1	−6	22	−67

$= x^2-6x+22+\frac{-67}{x+3}$

3. Evaluate $f(4)$ if $f(x)=x^3+7x-5$

	1	0	7	−5
4		4	16	92
	1	4	23	87

So $f(4)=87$

4. Evaluate $f(-1)$ if $f(x)=x^4-2x^3+5x^2+7x-11$

	1	−2	5	7	−11
−1		−1	3	−8	1
	1	−3	8	−1	−10

So $f(-1)=-10$

The Remainder Theorem: If a polynomial is divided by the factor $x - c$, the the value of the remainder r is equal to $f(c)$.

The Factor Theorem: For any polynomial $f(x)$:

(a) If $f(c) = 0$, then $(x - c)$ is a factor of $f(x)$. That is, $f(x) = (x - c)q(x)$ for some polynomial $q(x)$.

(b) If $(x - c)$ is a factor of $f(x)$, then $f(c) = 0$.

The Rational Root Theorem: If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is a polynomial with integer coefficients and $\frac{p}{q}$ is a rational zero of $f(x)$, then p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

Example: Let $f(x) = 4x^3 + 7x^2 + 26x - 7$. Using the rational root theorem, since $a_0 = -7$ and $a_n = 4$, we must have:

$p = \pm 1, \pm 7$ and $q = \pm 1, \pm 2, \pm 4$, hence $\frac{p}{q} = \pm 7, \pm\frac{7}{2}, \pm\frac{7}{4}, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}$ is a complete list of all the possible rational zeros of $f(x)$.

We will use synthetic division to show that $x = \frac{1}{4}$ is a zero of this polynomial function.

$$\begin{array}{c|cccc} & 4 & 7 & 26 & -7 \\ \frac{1}{4} & & 1 & 2 & 7 \\ \hline & 4 & 8 & 28 & 0 \end{array}$$

From this, we see that $f(x) = \left(x - \frac{1}{4}\right)\left(4x^2 + 8x + 28\right) = 4\left(x - \frac{1}{4}\right)\left(x^2 + 2x^2 + 7\right)$

To find the other zeros of this polynomial, we can apply the quadratic formula to the remaining quadratic polynomial $y = x^2 + 2x^2 + 7$.

Then $x = \dfrac{-2 \pm \sqrt{4 - 4(1)(7)}}{2(1)} = \dfrac{-2 \pm \sqrt{-24}}{2} = -1 \pm i\sqrt{6}$. Thus $x = -1 + i\sqrt{6}$ and $x = -1 - i\sqrt{6}$ are zeros of this polynomial.

To verify this, suppose we multiply the related factors:

$(x+1-i\sqrt{6})(x+1+i\sqrt{6}) = x^2 + x - i\sqrt{6}x + x + 1 - i\sqrt{6} + i\sqrt{6} + i\sqrt{6} - i^2 \cdot 6 = x^2 + 2x + 1 - 6i^2 = x^2 + 2x + 1 + 6 = x^2 + 2x + 7$

Hence the zeros of this polynomial are $x = \frac{1}{4}$, $x = -1 + i\sqrt{6}$, and $x = -1 - i\sqrt{6}$.

Notes:

1. If $f(x)$ is a polynomial of degree n, then, counting multiple roots separately, $f(x)$ will have n roots.

2. Complex roots always come in pairs. That is, if $a + bi$ is a root of a polynomial with real coefficients, then the conjugate $a - bi$ is also a root of the polynomial.