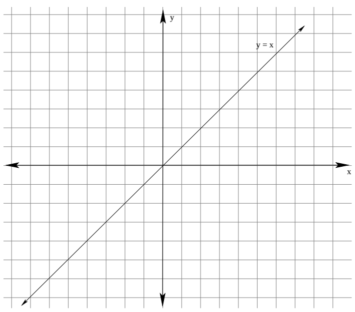
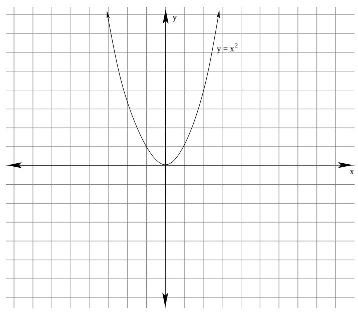
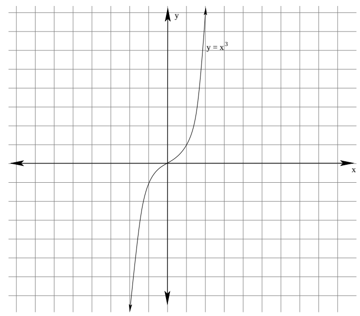
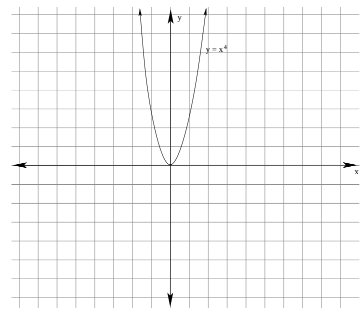
Math 127
Graphing Polynomial Functions Handout

Definition: A **polynomial function** is any function whose correspondence can be written in the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with $a_n \neq 0$.

Graphing Polynomial Functions:

Step 1: Determine the "End Behavior" of the polynomial $f(x)$.

Consider the following basic polynomial graphs:

Notice that there are four possibilities:
If a polynomial $f(x)$ has leading term ax^n with n even, then both "ends" of the graph point the **same direction**.
If $a > 0$, both ends point "upward". If $a < 0$, both ends point "downward".
If a polynomial $f(x)$ has leading term ax^n with n odd, then "ends" of the graph point in **opposite** directions.
If $a > 0$, the left end points "downward" and the right end points "upward".
If $a < 0$, the the left end points "upward" and the right end points "downward".

Examples:

1. Suppose $f(x) = 4x^7 - 3x^5 + 12x^4 - x^2 - 17x + 38$. The leading term is $4x^7$. Notice that this term has a positive coefficient (4) and an odd degree (7). Therefore, the left end points "downward" and the right end points "upward".

2. Suppose $f(x) = -7x^6 + 13x^5 + 2x^4 - 5x^3 - 17x^2 + 18x + 12$. The leading term is $-7x^6$. Notice that this term has a negative coefficient (-7) and an even degree (6). Therefore, both ends point "downward".

Step 2: Find and classify the x-intercepts (or **zeros**) of the polynomial $f(x)$.

The x-intercepts of a polynomial $f(x)$ correspond to the **roots** of the polynomial equation $f(x) = 0$. We generally find solutions to polynomial equations by factoring, or, if we can get down to a degree 2 polynomial, by using the quadratic equation.

When a polynomial equation is in factored form, we can find solutions by looking at the factors that occur.
We can also see the **multiplicity** of a solution.

For example, if $f(x) = x^3(x-1)(x+2)(x-3)^2$, then we see that the related function $f(x)$ has four zeros: $x = 0$, $x = 1$, $x = -2$, and $x = 3$.

By looking at the number of times each related factor occurs, we see that $x = 0$ is a zero with multiplicity 3, $x = 1$ and $x = -2$ both have multiplicity 1 since their related factors only occur once, and $x = 3$ is a zero of multiplicity 2 since its related factor $(x-3)$ occurs twice in the factored form of $f(x)$.

The reason that it is important to take note of the multiplicity of each zero is that the behavior of the graph at each zero depends on the multiplicity of the zero.

If $x = a$ is a zero with **odd** multiplicity, that the graph of $f(x)$ *crosses* the x-axis at the point $(a, 0)$. If $x = a$ is a zero with **even** multiplicity, that the graph of $f(x)$ *touches* the x-axis and *turns around* at the point $(a, 0)$.

Recall that a polynomial with degree n has at most n different zeros and at most $n - 1$ turning points.

Step 3: Find the y-intercept of $f(x)$.

The y-intercept of the polynomial $f(x)$ occurs at the point $(0, f(0))$. To find this intercept, we compute $f(0)$.

Step 4: Check to see if $f(x)$ has symmetry.

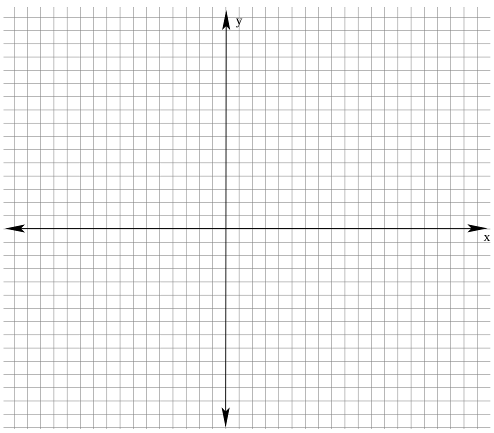
Recall that a function $f(x)$ is **even** if $f(-x) = f(x)$. In this case, the function $f(x)$ is symmetric with respect to the y-axis.

Similarly, a function $f(x)$ is **odd** if $f(-x) = -f(x)$. In this case, the function $f(x)$ is symmetric with respect to the origin.

Taking the time to check a polynomial for symmetry is helpful, since if we know the polynomial is symmetric with respect to either the y-axis or the origin, we can use this to fund extra points on the graph more easily.

Step 5: Use all the previously gathered information together with a few extra points as needed to sketch the graph of $f(x)$.

Example 1: Use the procedure described above to graph the polynomial $f(x) = x^4 + 2x^3 - 3x^2$



Example 2: Use the procedure described above to graph the polynomial $f(x) = (x-1)(x+2)^2(x-3)$