

Math 127
Exam 1 Practice Problems

1. Express the following number in scientific notation: .0001093

$$1.093 \times 10^{-4}$$

2. Express the following number in ordinary decimal notation: 4.03267×10^4

$$40326.7$$

3. True or False:

(a) $(a + b)c = ac + bc$

Solution: True (This is one of the forms of the distributive property).

(b) If $ab = 1$, then either $a = 1$ or $b = 1$ or both a and b equal 1

Solution: False. For example, if $a = \frac{1}{2}$ and $b = 2$, then $ab = 1$.

(c) $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$

Solution: False. For example, if $a = c = 3$, and $b = d = 1$, then $\frac{a}{b} + \frac{c}{d} = \frac{3}{1} + \frac{3}{1} = 6$, while $\frac{a+c}{b+d} = \frac{3+3}{1+1} = \frac{6}{2} = 3$.

(d) $\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$

Solution: True

(e) $5^{\frac{1}{2}} = \frac{1}{5^2}$

Solution: False. $5^{\frac{1}{2}} = \sqrt{5}$, while $\frac{1}{5^2} = \frac{1}{25}$.

(f) $(a + b)^2 = a^2 + b^2$

Solution: False. $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$.

(g) $x = 0$ is a solution to the equation $\frac{x^2}{x} = 0$

Solution: False. Division by zero is not defined for real numbers.

4. Simplify the following:

(a) $\left(\frac{3}{4}\right)^{-2}$

Solution:

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

(b) $8^{\frac{4}{3}}$

Solution:

$$8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$$

(c) $\left(\frac{y^{12}}{25z^4}\right)^{-\frac{3}{2}}$

$$\left(\frac{y^{12}}{25z^4}\right)^{-\frac{3}{2}} = \left(\frac{25z^4}{y^{12}}\right)^{\frac{3}{2}} = \left(\sqrt{\frac{25z^4}{y^{12}}}\right)^3 = \left(\frac{5z^2}{y^6}\right)^3 = \frac{125z^6}{y^{18}}$$

(d) $\sqrt[5]{32x^{11}y^{14}z^8}$

Solution:

$$\sqrt[5]{32x^{11}y^{14}z^8} = \sqrt[5]{32x^{10}xy^{10}y^4z^5z^3} = 2x^2y^2z\sqrt[5]{xy^4z^3}$$

$$(e) \left(\frac{(5xyz)^2 z^{-2}}{2x^{-2}y^2 z^{-4}} \right)^{-1}$$

Solution:

$$\left(\frac{(5xyz)^2 z^{-2}}{2x^{-2}y^2 z^{-4}} \right)^{-1} = \left(\frac{25x^2 y^2 z^2 z^{-2}}{2x^{-2}y^2 z^{-4}} \right)^{-1} = \left(\frac{25x^{2+2}y^{2-2}z^{2-2+4}}{2} \right)^{-1} \left(\frac{25x^4 z^4}{2} \right)^{-1} = \frac{2}{25x^4 z^4}$$

5. Rationalize the denominator in the following expressions:

$$(a) \frac{3x}{\sqrt[3]{x}}$$

Solution:

$$\frac{3x}{\sqrt[3]{x}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{3x\sqrt[3]{x^2}}{x} = 3\sqrt[3]{x^2}$$

$$(b) \frac{2x+3}{\sqrt{2x-1}}$$

Solution:

$$\frac{2x+3}{\sqrt{2x-1}} \cdot \frac{\sqrt{2x+1}}{\sqrt{2x+1}} = \frac{(2x+3)\sqrt{2x+1}}{(2x-1)}$$

6. Perform the indicated operations and simplify:

$$(a) 3(2x^3 - x^2 + 5x) - 2x(3x^3 - 2x^2 + 5x - 3)$$

Solution:

$$3(2x^3 - x^2 + 5x) - 2x(3x^3 - 2x^2 + 5x - 3) = 6x^3 - 3x^2 + 15x - 6x^4 + 4x^3 - 10x^2 + 6x = -6x^4 + 10x^3 - 13x^2 + 21x$$

$$(b) (2x^2 + 3x - 2)(x - 2)$$

Solution:

$$(2x^2 + 3x - 2)(x - 2) = 2x^3 + 3x^2 - 2x - 4x^2 - 6x + 4 = 2x^3 - x^2 - 8x + 4$$

$$(c) (2x+1)^3$$

Solution:

$$(2x+1)^3 = (2x+1)(2x+1)(2x+1) = (4x^2+4x+1)(2x+1) = 8x^3+4x^2+8x^2+4x+2x+1 = 8x^3+12x^2+6x+1, \text{ or, use the expansion formula for cubes, which yields the same result.}$$

$$(d) (x^{\frac{1}{3}} + y^{\frac{1}{3}})(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$$

Solution:

$$= x + x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} + y = x + y$$

7. Factor each of the following expressions completely:

$$(a) 2x^2 + x - 6$$

Solution: $(2x-3)(x+2)$

$$(b) 50x^2 + 45x - 18$$

Solution:

$$(50 \cdot -18 = -900 = -1 \cdot 10 \cdot 10 \cdot 9 = -1 \cdot 5 \cdot 5 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 60 \cdot -15) \\ = 50x^2 + 60x - 15x - 18 = 10x(5x+6) - 3(5x+6) = (5x+6)(10x-3)$$

$$(c) 9x^2 - 49y^6$$

Solution: $(3x-7y^3)(3x+7y^3)$

$$(d) 8x^3 - y^3$$

Solution: $(2x-y)(4x^2+2xy+y^2)$

$$(e) 6x^3y - 27x^2y - 15xy$$

Solution: $(3xy)(2x^2-9x-5) = (3xy)(2x+1)(x-5)$

$$(f) 3x^3 + x^2 - 3x - 1$$

Solution: $x^2(3x+1) - 1(3x+1) = (x^2-1)(3x+1) = (x+1)(x-1)(3x+1)$

(g) $x^6 - 1$

Solution:

$$\begin{aligned}x^6 - 1 &= (x^2)^3 - (1)^3, \text{ so by the difference of cubes factoring formula:} \\ &= (x^2 - 1)(x^4 + x^2 + 1) \\ &= (x + 1)(x - 1)(x^4 + x^2 + 1)\end{aligned}$$

8. Simplify the following expressions:

(a) $\frac{3x^2 - 10x + 3}{x^2 - 1} \cdot \frac{x^2 + x - 2}{x^2 - 9}$

Solution:

$$\frac{3x^2 - 10x + 3}{x^2 - 1} \cdot \frac{x^2 + x - 2}{x^2 - 9} = \frac{(3x - 1)(x - 3)}{(x + 1)(x - 1)} \cdot \frac{(x + 2)(x - 1)}{(x + 3)(x - 3)} = \frac{(3x - 1)(x + 2)}{(x + 1)(x + 3)}$$

(b) $\frac{2x^2 + 4}{2x^2 + 7x - 4} - \frac{x - 1}{x + 4}$

Solution:

$$\begin{aligned}\frac{2x^2 + 4}{2x^2 + 7x - 4} - \frac{x - 1}{x + 4} &= \frac{2x^2 + 4}{(2x - 1)(x + 4)} - \frac{x - 1}{x + 4} = \frac{2x^2 + 4}{(2x - 1)(x + 4)} - \frac{(2x - 1)(x - 1)}{(2x - 1)(x + 4)} \\ &= \frac{2x^2 + 4 - (2x^2 - 3x + 1)}{(2x - 1)(x + 4)} = \frac{2x^2 + 4 - 2x^2 + 3x - 1}{(2x - 1)(x + 4)} = \frac{3x + 3}{(2x - 1)(x + 4)} = \frac{3(x + 1)}{(2x - 1)(x + 4)}\end{aligned}$$

(c) $\frac{\frac{1}{x} + \frac{3}{x-2}}{\frac{4}{x-1} - \frac{2}{x-2}}$

Solution:

$$\begin{aligned}\frac{\frac{1}{x} + \frac{3}{x-2}}{\frac{4}{x-1} - \frac{2}{x-2}} \cdot \frac{x(x-1)(x-2)}{x(x-1)(x-2)} \\ &= \frac{(x-1)(x-2) + 3(x)(x-1)}{4(x)(x-2) - 2(x)(x-1)} = \frac{(x-1)(x-2+3x)}{x[4(x-2) - 2(x-1)]} \\ &= \frac{(x-1)(4x-2)}{x[4x-8-2x+2]} = \frac{2(x-1)(2x-1)}{x(2x-6)} = \frac{2(x-1)(2x-1)}{2x(x-3)} = \frac{(x-1)(2x-1)}{x(x-3)}\end{aligned}$$

(d) $\frac{\frac{3}{2x+2h+1} - \frac{3}{2x+1}}{h}$

Solution:

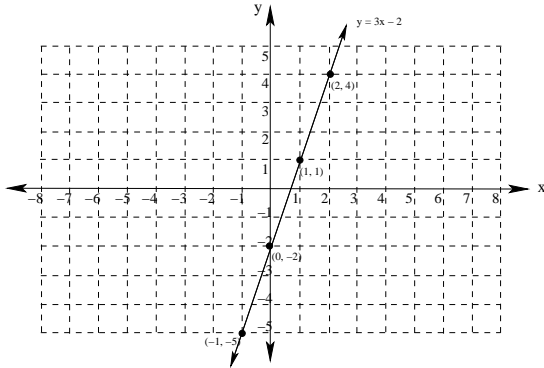
(e) $\frac{\frac{3}{2x+2h+1} - \frac{3}{2x+1}}{h} = \frac{\frac{3(2x+1)}{(2x+2h+1)(2x+1)} - \frac{3(2x+2h+1)}{(2x+1)(2x+2h+1)}}{h}$

(f) $= \frac{(6x+3) - (6x+6h+3)}{(2x+2h+1)(2x+1)} \cdot \frac{1}{h} = \frac{-6h}{(2x+1)(2x+2h+1)(h)}$

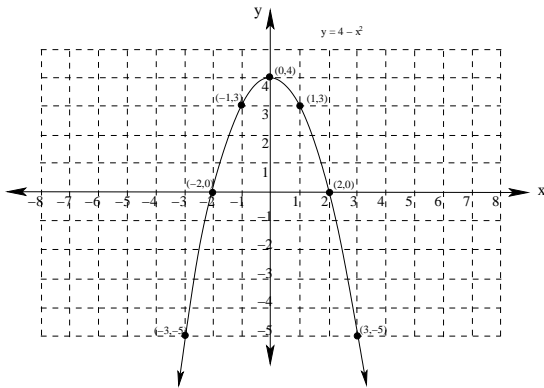
(g) $= \frac{-6}{(2x+1)(2x+2h+1)}$

9. Sketch the graphs of the following equations:

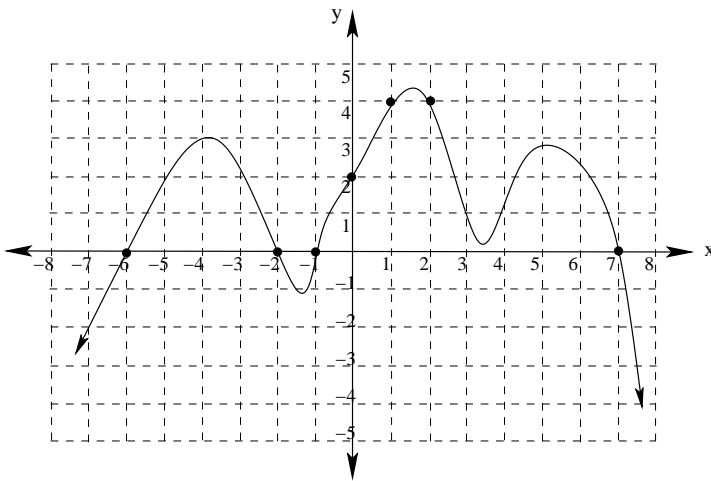
(a) $y = 3x - 2$



(b) $y = 4 - x^2$



10. Based on the graph given below:



(a) Find the coordinates of all x intercepts.

From the graph, we see that the x -intercepts are at the points: $(-6, 0)$, $(-2, 0)$, $(-1, 0)$ and $(7, 0)$

(b) Find the coordinates of all y intercepts.

From the graph, we see that the y -intercept is at the point: $(0, 2)$

(c) Find the x -value(s) when $y = 4$

From the graph, we see that the x -values corresponding to $y = 4$ are $x = 1$ and $x = 2$.