Math 127 **Name:**______________________________

Chapter 3 Quiz

06/24/2013

1. (5 points) Let $f(x) = -4x^2 + 4x + 3$.

(a) Find the vertex and axis of symmetry.

We complete the square:

$f(x) = -4(x^2 - x + ?) + 3 - ?$

$f(x) = -4\left(x^2 - x + \left(-\frac{1}{2}\right)^2\right) + 3 - (-4) \cdot \frac{1}{4}$

$f(x) = -4\left(x - \left(\frac{1}{2}\right)^2\right) + 3 + 1$

$f(x) = -4\left(x - \left(\frac{1}{2}\right)^2\right) + 4$

Thus the vertex is at $\left(\frac{1}{2}, 4\right)$ and the axis of symmetry is $x = \frac{1}{2}$

(b) Find the x-intercepts and y-intercepts of $f(x)$.

To find the x-intercepts, notice that if $-4x^2+4x+3 = 0$, then $4x^2 - 4x - 3 = 0$

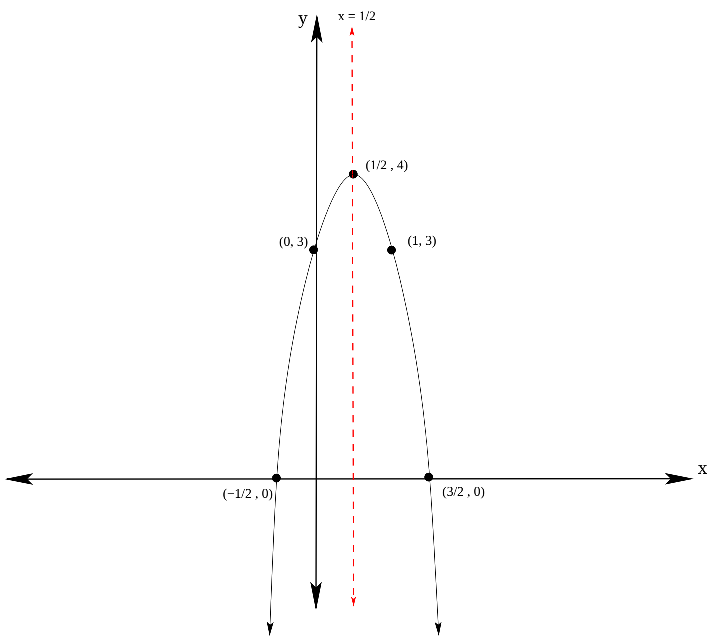
This factors as $(2x-3)(2x+1) = 0$, so the x-intercepts occur when $x = \frac{3}{2}$ and $x = -\frac{1}{2}$

Therefore, the x-intercepts are $\left(\frac{3}{2}, 0\right)$ and $\left(-\frac{1}{2}, 0\right)$

The y-intercept occurs when $f(0) = 3$, or at the point $(0, 3)$

Notice that, using symmetry, $(1, 3)$ is another point on the graph.

(c) Graph $f(x)$ on the axes provided, labeling all key features of the graph.



2. (5 points) Let $f(x) = (x+1)(x-2)^2(x-3)^2$

(a) Find the leading term of $f(x)$ and use it to determine the end behavior of the graph of $f(x)$.

Collecting the highest order of terms from the product above, we see that the leading term is $x \cdot x^2 \cdot x^2 = x^5$

Therefore, the graph approaches $-\infty$ on the left and it approaches $+\infty$ on the right.

(b) Find the zeros of $f(x)$. Also find the multiplicity of each zero and state whether the graph crosses or touches the x-axis at each zero.

Since the polynomial is in factored form, we can read off the zeros:

$x = -1$ has multiplicity 1, so the graph crosses the x-axis there.

$x = 2$ has multiplicity 2, so the graph touches the x-axis and turns around.

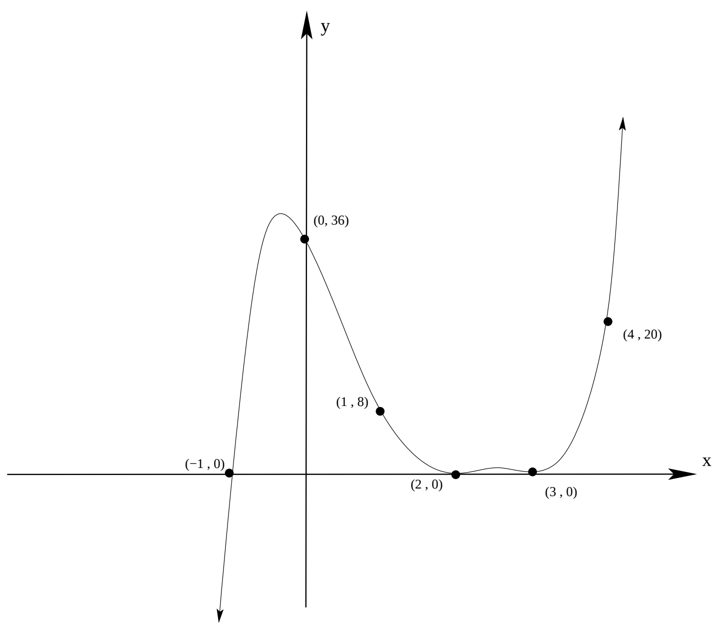
$x = 3$ also has multiplicity 2, so the graph touches the x-axis and turns around.

(c) Find the y-intercept of $f(x)$ and at least one additional point on the graph of $f(x)$.

The y-intercept occurs at $f(0) = (0+1)(0-2)^2(0-3)^2 = (1)(4)(9) = 36$

Other points: $f(1) = (2)(-1)^2(-2)^2 = 2(1)(4) = 8$ and $f(4) = (5)(2)^2(1)^2 = 20$

(d) Graph $f(x)$ on the axes provided, labeling all key features of the graph.



3. (5 points) Let $g(x) = 3x^3 + 2x^2 - 13x + 4$

(a) Use the Rational Root Theorem to list all possible rational zeros for $g(x)$.

$a_0 = -4$, so $p = \pm 1, \pm 2, \pm 4$

$a_n = 3$, so $q = \pm 1, \pm 3$

Then the possible rational roots are: $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

(b) Use synthetic division to show that $\dfrac{1}{3}$ is a zero of $g(x)$.

	3	2	-13	4
$\frac{1}{3}$		1	1	-4
	3	3	-12	0

(c) Find **all** solutions to the equation $g(x) = 0$.

From above, we see that $g(x) = \left(x - \frac{1}{3}\right)(3x^2 + 3x - 12) = 3\left(x - \frac{1}{3}\right)(x^2 + x - 4)$

Then we look at $x^2 + x - 4 = 0$ to find the remaining zeros. Applying the quadratic formula,

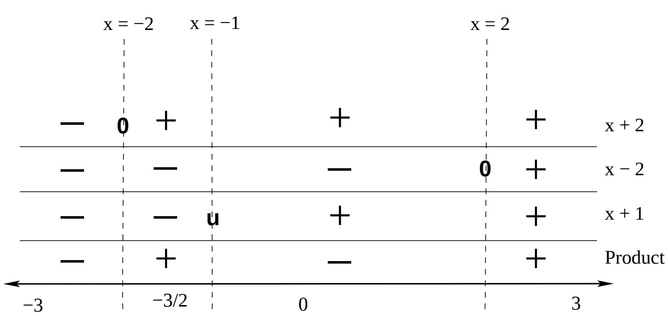
$$x = \frac{-1 \pm \sqrt{1 - 4(1)(-4)}}{2(1)} = \frac{-1 \pm \sqrt{17}}{2}$$

Thus the solutions are $x = \frac{1}{3}$, $x = -\frac{1}{2} + \frac{\sqrt{17}}{2}$, and $x = -\frac{1}{2} - \frac{\sqrt{17}}{2}$.

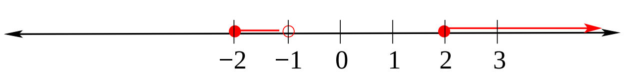
4. (5 points) Find all solutions to the inequality $\dfrac{x^2 - 4}{x + 1} \geq 0$. Graph your solution on a number line. Also give the solution in interval notation.

First, we factor this expression, obtainiing $\frac{(x-2)(x+2)}{x+1} \geq 0$

The key values for this inequality are $x = 2$, $x = -2$, and $x = -1$. We now make a sign chart:



From this, we see that the solution set is:



Or, in interval notation, $[-2, -1) \cup [2, \infty)$.