Math 127 - Exam 1 **Name:**______________________________
Version 1
05/29/2013

You MUST show appropriate work to receive credit

1. (3 points) Express the product $\left(5.1 \times 10^7\right)\left(2 \times 10^{-3}\right)$ in scientific notation.

   $\left(5.1 \times 10^7\right)\left(2 \times 10^{-3}\right) = 10.2 \times 10^{7-3} = 10.2 \times 10^4 = 1.02 \times 10^5$

2. (2 points) Rewrite the expression $|5 - \sqrt{11}|$ without using the absolute value symbol.

   Since $5 > \sqrt{11}$, $|5 - \sqrt{11}| = 5 - \sqrt{11}$.

3. (4 points each) True or False (Include a *brief* justification of your answer):

   (a) $\sqrt{a^2 + b^2} = a + b$

   False.

   Notice that if $a = 2$ and $b = 3$, then $\sqrt{a^2 + b^2} = \sqrt{4 + 9} = \sqrt{13}$ while $a + b = 2 + 3 = 5$.

   (b) $\dfrac{a}{b} + \dfrac{c}{b} = \dfrac{a + c}{b}$

   True.

   Notice that the fractions in this expression have a common denominator, so we can add the numerators and place them over the common denominator.

   (c) $2^{-3} = -8$

   False.

   Notice that $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$.

   (d) $\{b, c, d\} \cap \{d, e, f\} = \{b, c, d, e, f\}$

   False.

   Since we are finding the intersection of these sets, $\{b, c, d\} \cap \{d, e, f\} = \{d\}$

4. (3 points each) Simplify and/or evaluate each of the following.

   (a) $\dfrac{3(5^2 - 8) - 3\left[2 + (-3)^2\right]}{-4^2 \div (7 - 3)}$

   $= \dfrac{3(25 - 8) - 3\,[2 + 9]}{-16 \div 4} = \dfrac{3(17) - 3\,[11]}{-4} = \dfrac{51 - 33}{-4} = \dfrac{18}{-4} = -\dfrac{9}{2}.$

   (b) $x^3y - 5x^2y + 4x - 2$ if $x = -1$ and $y = 2$

   $= (-1)^3(2) - 5(-1)^2(2) + 4(-1) - 2 = (-1)(2) - (5)(1)(2) - 4 - 2 = -2 - 10 - 4 - 2 = -18.$

5. Use properties of exponents and radicals to simplify the following expression. Your answer should have no negative exponents. Assume all variables represent nonnegative numbers.

(a) (5 points) $\left(x^{\frac{1}{2}}\right)^3 \cdot x^5$

$$\left(x^{\frac{1}{2}}\right)^3 \cdot x^5 = x^{\frac{3}{2}} \cdot x^5 = x^{\frac{3}{2}+5} = x^{\frac{3}{2}+\frac{10}{2}} = x^{\frac{13}{2}}.$$

(b) (5 points) $\left(\dfrac{10x^{-2}y^3}{2^{-2}x^3y^{-4}}\right)^2$

$$\left(\frac{10x^{-2}y^3}{2^{-2}x^3y^{-4}}\right)^2 = \left(\frac{10 \cdot 2^2y^3y^4}{x^3 \cdot x^2}\right)^2 = \left(\frac{40y^7}{x^5}\right)^2 = \frac{40^2y^{14}}{x^{10}} = \frac{1600y^{14}}{x^{10}}.$$

(c) (5 points) $\sqrt[4]{16x^6y^7z^8}$

$$\sqrt[4]{16x^6y^7z^8} = \sqrt[4]{2^4x^4x^2x^2y^4y^3y^4z^4} = 2xyz^2\sqrt[4]{x^2y^3}.$$

6. Rationalize all denominators and simplify. Assume all variables represent positive values.

(a) (5 points) $\dfrac{5x^2}{\sqrt[3]{9xy^2}}$

$$\frac{5x^2}{\sqrt[3]{9xy^2}} \cdot \frac{\sqrt[3]{3x^2y}}{\sqrt[3]{3x^2y}} = \frac{5x^2\sqrt[3]{3x^2y}}{\sqrt[3]{27x^3y^3}} = \frac{5x^2\sqrt[3]{3x^2y}}{3xy} = \frac{5x\sqrt[3]{3x^2y}}{3y}$$

(b) (5 points) $\dfrac{\sqrt{3}}{x-\sqrt{3}}$

$$\frac{\sqrt{3}}{x-\sqrt{3}} \cdot \frac{x+\sqrt{3}}{x+\sqrt{3}} = \frac{x\sqrt{3}+3}{x^2+x\sqrt{3}-x\sqrt{3}-3} = \frac{x\sqrt{3}+3}{x^2-3}.$$

7. (5 points) Simplify the following expression:

$(2x-5y)^2-(2x+5y)^2$

$$(2x-5y)^2-(2x+5y)^2 = (4x^2-10xy-10xy+25y^2)-(4x^2+10xy+10xy+25y^2)$$

$$= 4x^2-20xy+25y^2-(4x^2+20xy+25y^2)$$

$$= 4x^2-20xy+25y^2-4x^2-20xy-25y^2$$

$$= -40xy.$$

8. (5 points each) Factor each of the following *completely.* Box your answers.

(a) $12x^2 + 5x - 3$

Using the "ac-split", notice that
$ac = (12)(-3) = -36 = (9)(-4)$, and $9 + (-4) = 5$.
Then $12x^2 + 5x - 3 = 12x^2 + 9x - 4x - 4$
$= 3x(4x + 3) - 1(4x + 3)$
$= (4x + 3)(3x - 1)$.

(b) $x^3 + 3x^2 - 4x - 12$

Using factoring by grouping,
$x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3)$
$= (x + 3)(x^2 - 4) = (x + 3)(x + 2)(x - 2)$.

(c) $x^3 - 27$

Notice that this is a difference of cubes: $(x^3 - 3^3)$.
Therefore, using the special formula for a difference of cubes:

$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$

(d) $x^4 - 1$

Notice that this is a difference of squares,
so $x^4 - 1 = (x^2 + 1)(x^2 - 1)$.

The second term is also a difference of squares,
so this factors further as $(x^2 + 1)(x + 1)(x - 1)$.

9. (7 points each) Perform the operations indicated and simplify each of the following as much as possible. Your answer should be completely reduced and should contain no complex fractions.

(a) $\dfrac{x^2 - x - 12}{x^2 - 4} \div \dfrac{x^2 - 2x - 8}{x^2 + x - 6}$

Factoring, we have $\dfrac{(x+3)(x-4)}{(x+2)(x-2)} \div \dfrac{(x-4)(x+2)}{(x+3)(x-2)}$

Next, we change from multiplication to division: $\dfrac{(x+3)(x-4)}{(x+2)(x-2)} \cdot \dfrac{(x+3)(x-2)}{(x-4)(x+2)}$

Now, we divide out common factors and combine into a single fraction: $\dfrac{(x+3)(x+3)}{(x+2)(x+2)}$.

Thus our final simplified answer is: $\dfrac{(x+3)^2}{(x+2)^2}$

(b) $\dfrac{8x + 12}{x^2 + 3x - 10} + \dfrac{x + 1}{x + 5}$

First, we factor in order to obtain: $\dfrac{8x + 12}{(x+5)(x-2)} + \dfrac{x+1}{x+5}$. Notice that the LCD is: $(x + 5)(x - 2)$

Multiplying to get all terms over the LCD gives:

$$\frac{8x + 12}{(x+5)(x-2)} + \frac{x+1}{x+5} \cdot \frac{(x-2)}{(x-2)} = \frac{8x + 12 + (x+1)(x-2)}{(x+5)(x-2)}.$$

Simplifying, we get:

$$\frac{8x + 12 + x^2 - x - 2}{(x+5)(x-2)} = \frac{x^2 + 7x + 10}{(x+5)(x-2)}$$

$$= \frac{(x+5)(x+2)}{(x+5)(x-2)} = \frac{x+2}{x-2}.$$

(c) $\dfrac{\frac{1}{x}+\frac{1}{3}}{1-\frac{2}{x}}$

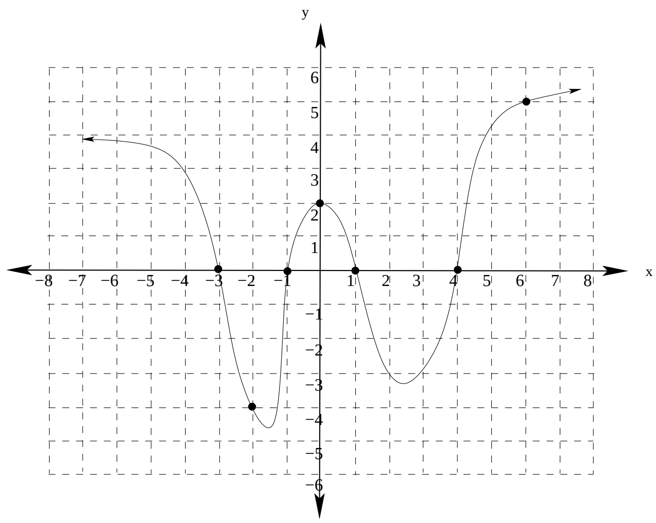
First, we multiply both the numerator and the denominator by the LCD: $3x$ and simplify:

$$\frac{\frac{1}{x}+\frac{1}{3}}{1-\frac{2}{x}} \cdot \frac{\frac{3x}{1}}{\frac{3x}{1}} = \frac{\frac{3x}{x}+\frac{3x}{3}}{3x-\frac{6x}{x}}$$

Simplifying this yields:

$$\frac{3+x}{3x-6} = \frac{x+3}{3(x-2)}$$

10. (2 points each) Answer each of the following questions based on the graph shown below:



(a) List the coordinates of the x-intercept(s) of this graph.

$(-3,0)$, $(-1,0)$, $(1,0)$ and $(4,0)$

(b) List the coordinates of the y-intercept(s) of this graph.

$(0,2)$

(c) Give the x coordinate of the point(s) where $y=5$

$x=6$

(d) Give the y coordinate of the point(s) where $x=-2$

$y=-4$