Math 127 - Exam 2 **Name:**______________________________
Version 1
06/05/2013

You MUST show appropriate work to receive credit

1. (4 points each) True or False (Include a *brief* justification of your answer):

   (a) The equation $3x = x$ has no solutions.

   False. Careful inspection (or actually solving this equation) shows that $x = 0$ is a solution.

   (b) The equation $|5 - 2x| = 0$ has two solutions.

   False. While most absolute value equations have two solutions, this one only has a single solution. This is due to the zero on the right hand side of the equation – both positive and negative cases yield the same solution for this example.

2. (6 points) Use completing the square to solve the quadratic equation $2x^2 - 6x - 3 = 0$.

   $2x^2 - 6x = 3$ [move the constant to the right hand side]
   $x^2 - 3x = \frac{3}{2}$ [divide both sides by 2]
   $x^2 - 3x + \left(\frac{3}{2}\right)^2 = \frac{3}{2} + \left(\frac{3}{2}\right)^2$ [add the appropriate constant]
   $(x - \frac{3}{2})^2 = \frac{3}{2} + \frac{9}{4}$ [factor the left hand side]
   $(x - \frac{3}{2})^2 = \frac{6}{4} + \frac{9}{4}$
   $(x - \frac{3}{2})^2 = \frac{15}{4}$ [combine constants on the right hand side]
   $x - \dfrac{3}{2} = \pm\sqrt{\dfrac{15}{4}}$ [take the square root of both sides]
   $x = \dfrac{3}{2} \pm \dfrac{\sqrt{15}}{\sqrt{4}}$ [simplify]
   $x = \dfrac{3}{2} \pm \dfrac{\sqrt{15}}{2}$

3. Perform the indicated operations and express your answer in the form $a + bi$:

   (a) (4 points) $(5 - 3i)^2$

   $(5 - 3i)^2 = (5 - 3i)(5 - 3i) = 25 - 15i - 15i + 9i^2 = 25 - 30i - 9 = 16 - 30i$

   (b) (4 points) $\dfrac{5 - 2i}{7 - i}$

   $$\frac{5 - 2i}{7 - i} \cdot \frac{7 + i}{7 + i} = \frac{35 - 14i + 5i - 2i^2}{49 + 7i - 7i - i^2} = \frac{35 + 2 - 9i}{49 + 1} = \frac{37 - 9i}{50} = \frac{37}{50} + \frac{-9}{50}i$$

4. (6 points each) Find **all** solutions to the following equations:

(a) $\dfrac{3}{5} + \dfrac{x}{2} = \dfrac{4-x}{7}$

We multiply by the LCD, $70 = (5)(2)(7)$: $(5)(2)(7) \cdot \left[\dfrac{7}{10} - \dfrac{x}{3} = \dfrac{2x-5}{2}\right]$

$(7)(2)(3) + (5)(7)x = (5)(2)(4-x)$, or $42 + 35x = 40 - 10x$.

Then $45x = -2$, so $x = -\frac{2}{45}$.

(b) $\dfrac{3x+1}{6x-2} = \dfrac{2x+5}{4x-13}$

We multiply by the LCD, $(6x-2)(4x-13)$: $[(6x-2)(4x-13)]\dfrac{3x+1}{6x-2} = [(6x-2)(4x-13)]\dfrac{2x+5}{4x-13}$

$(4x-13)(3x+1) = (6x-2)(2x+5)$, or $12x^2 - 39x + 4x - 13 = 12x^2 - 4x + 30x - 10$

then $12x^2 - 35x - 13 = 12x^2 + 26x - 10$, so, combining terms, $-3 = 61x$, thus $x = -\frac{3}{61}$.

(c) $5x^2 - x = -1$

We immediately rearrange the terms to put this quadratic into standard form: $5x^2 - x + 1 = 0$

Since this quadratic equation does not factor, the most straightforward way to solve this is to use the quadratic formula.

Notice that $a = 5$, $b = -1$, and $c = 1$.

Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{1 \pm \sqrt{1 - 4(5)(1)}}{2(5)} = \dfrac{1 \pm \sqrt{1-20}}{10}$

So $x = \dfrac{1 \pm \sqrt{-19}}{10} = \dfrac{1 \pm i\sqrt{19}}{10} = \dfrac{1}{10} \pm \dfrac{i\sqrt{19}}{10}$

(d) $x^3 - 3x^2 = 4x - 12$

We again begin by rearranging the terms to get everything on the same side: $x^3 - 3x^2 - 4x + 12 = 0$

Notice that this polynomial factors by grouping: $x^2(x-3) - 4(x-3) = 0$, or $(2-3)(x^2-4) = 0$

Using the zero product property to split into 2 cases, we see that either $x - 3 = 0$, so $x = 3$
or $x^2 - 4 = 0$, in which case, $x^2 = 4$, so $x = \pm\sqrt{4} = \pm 2$.
Then $x = 3$ or $x = 2$ or $x = -2$.

(e) $\sqrt{3x+1} - \sqrt{x+4} = 1$

This problem ends up being a bit nicer if we rearrange to get: $\sqrt{3x+1} = 1 + \sqrt{x+4}$

Squaring both sides, we get: $\left(\sqrt{3x+1}\right)^2 = \left(1 + \sqrt{x+4}\right)^2$

which simplifies to give: $3x + 1 = 1 + 2\sqrt{x+4} + x + 4$ or $3x + 1 = x + 5 + \sqrt{x+4}$

Moving terms to isolate the remaining radical gives: $2x - 4 = 2\sqrt{x+4}$, or $x - 2 = \sqrt{x+4}$

Squaring again gives: $(x-2)^2 = x + 4$, or $x^2 - 4x + 4 = x + 4$

Then $x^2 - 5x = 0$, or $x(x-5) = 0$, so $x = 0$ or $x = 5$.

It is imperative that we check our answer, since the method of squaring can introduce extraneous solutions.

If $x = 0$, then $\sqrt{3x+1} - \sqrt{x+4} = \sqrt{1} - \sqrt{4} = 1 - 2 = -1$, so this solution does not check.

If $x = 5$, then $\sqrt{3x+1} - \sqrt{x+4} = \sqrt{16} - \sqrt{9} = 4 - 3 = 1$, so this solution does check.

Hence this equation has one solution: $x = 5$

(f) $t^4 - t^2 - 12 = 0$

Here, we can either factor this expression, or, to make it a bit easier, we can substitute using $u = t^2$ and $u^2 = t^4$.

This gives $u^2 - u - 12 = 0$, or $(u-4)(u+3) = 0$

Therefore, either $u = 4$ or $u = -3$

That is, either $t^2 = 4$ or $t^2 = -3$

But then either $t = \pm\sqrt{2}$ or $t = \pm\sqrt{-3} = \pm i\sqrt{3}$.

Notice that all 4 of these solutions check.

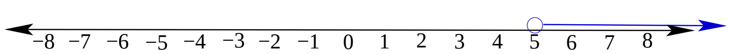
5. Solve the following inequalities. Graph your solution on a number line, and express your answer in interval notation.

(a) (6 points) $5x - (3x+1) < 3(x-2)$

Simplifying, we get $5x - 3x - 1 < 3x - 6$, or $2x - 1 < 3x - 6$

Moving terms to isolate x, we get $-x < 5$, or $x > 5$.

In interval notation, this is: $(5, \infty)$



(b) (6 points) $|3x - 2| - 2 \geq 4$

We first isolate the absolute value part of the inequality: $|3x - 2| \geq 6$

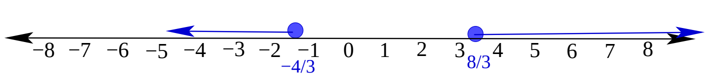
Since the constant term is positive, this inequality has solutions. We proceed by splitting into two cases.

Positive case: $3x - 2 \geq 6$ Negative case: $-(3x-2) \geq 6$ or $3x - 2 \leq -6$

Then $3x \geq 8$ or $3x \leq -4$

Thus $x \geq \dfrac{8}{3}$ or $x \leq -\frac{4}{3}$

Which, in interval notation gives: $(-\infty, -\frac{4}{3}] \cup [\frac{8}{3}, \infty)$



6. (8 points) A movie theater charges adults \$7 per ticket wile children pay \$4 per ticket. At one afternoon showing, 50 total tickets sold for \$218. How many adults bought tickets to the show? [You must use algebra and the complete problem solving process to get credit on this problem.]

   A careful reading of the situation described above suggests that this problem is best viewed as a "mixing problem". We will let x be the number of tickets sold to adults and y the number of tickets sold to children.

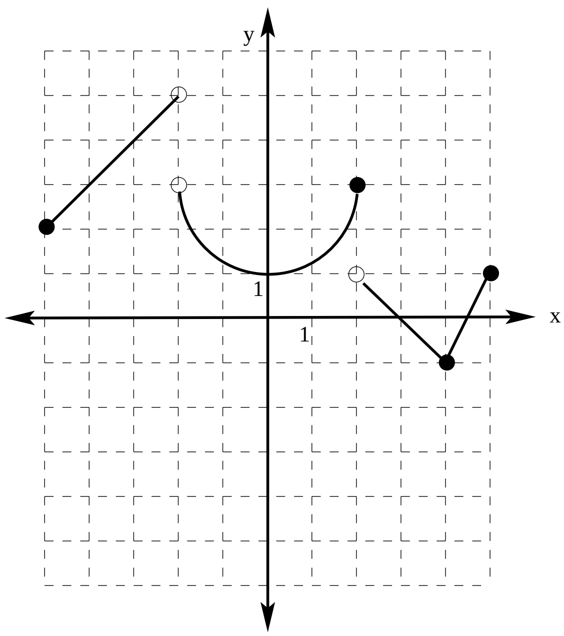
   From this, we obtain the equations $x + y = 50$ (50 total tickets are sold) and $7x + 4y = 218$ (a total \$218 was brought in from \$7 adult tickets and \$4 children's tickets)

   Since $x + y = 50$, then $y = 50 - x$, so we substitute to obtain: $7x + 4(50 - x) = 218$.

   Then $7x + 200 - 4x = 218$, or $3x = 18$. Hence $x = \frac{18}{3} = 6$. (We then also know that $y = 50 - 6 = 44$.

   Therefore, there were 6 adults who bought tickets to this show.

7. For the given graph of $f(x)$, find the following:



   (a) (2 points) $f(4)$

   $f(4) = -1$ (see the graph)

   (b) (2 points) x, when $f(x) = 3$

   $x = -4$ or $x = 2$

   (c) (2 points) The domain of f

   $[-5, -2) \cup (-2, 5]$

   (d) (2 points) The range of f

   $[-1, 5]$

   (e) (3 points) The intervals where f is increasing.
   $[5, -2)$; $[0, 2]$; $[4, 5]$.

8. Let $f(x) = x^2 - 3x$. Find and simplify the following:

(a) (2 points) $f(-1)$

$f(-1) = (-1)^2 - 3(-1) = 1 + 3 = 4.$

(b) (3 points) $f(a-2)$

$f(a-2) = (a-2)^2 - 3(a-2) = a^2 - 4a + 4 - 3a + 6 = a^2 - 7a + 10 = (a-5)(a-2).$

(c) (5 points) $\dfrac{f(a+h) - f(a)}{h}$

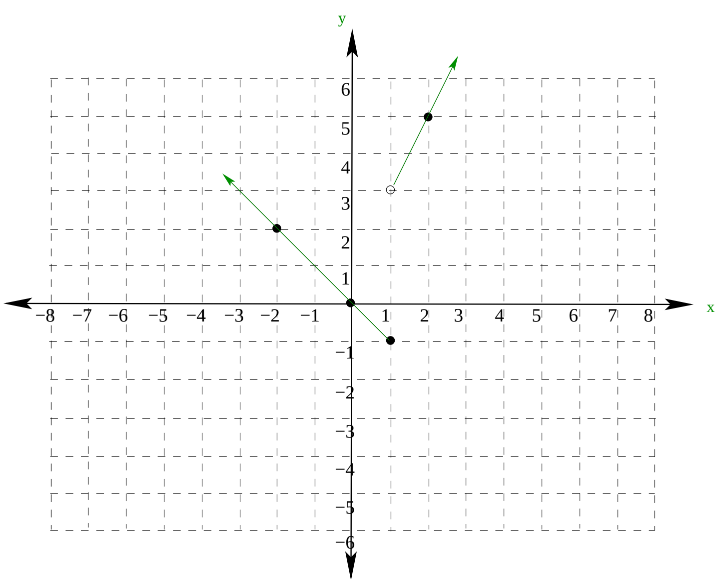
Notice that $f(a+h) = (a+h)^2 - 3(a+h) = a^2 + 2ah + h^2 - 3a - 3h.$

Therefore, $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(a^2 + 2ah + h^2 - 3a - 3h) - (a^2 - 3a)}{h} = \dfrac{a^2 + 2ah + h^2 - 3a - 3h - a^2 + 3a}{h}$

$= \dfrac{2ah + h^2 - 3h}{h} = \dfrac{h(2a + h - 3)}{h} = 2a + h - 3.$

9. (5 points) Draw the graph of the function $f(x) = \begin{cases} -x & \text{if } x \leq 1 \\ 2x + 1 & \text{if } x > 1 \end{cases}$.

Label your axes and at least 4 points on the graph.



x	$y = -x$
-2	2
-1	1
0	0
1	-1
x	$y = 2x + 1$
1	3
2	5
3	7

Extra Credit: (5 points) Use completing the square on the general quadratic polynomial $ax^2 + bx + c = 0$ to derive the quadratic formula

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac}{4a^2} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac+b^2}{4a^2}}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$