Math 127 - Exam 3 **Name:**____________________
6/11/2013
You MUST show appropriate work to receive credit

1. Given the points $A(3,-5)$ and $B(-1,3)$:

   (a) (5 points) Find $d(A,B)$ (if necessary, round your answer to 2 decimal places)

   $d(A,B)=\sqrt{(3-(-1))^2+(-5-3)^2}=\sqrt{4^2+(-8)^2}=\sqrt{16+64}=\sqrt{80}=4\sqrt{5}\approx 8.94$

   (b) (5 points) Find the midpoint of the line segment containing A and B.

   $$M=\left(\frac{3+(-1)}{2},\frac{-5+3}{2}\right)=\left(\frac{2}{2},\frac{-2}{2}\right)=(1,-1)$$

   (c) (5 points) Find the equation for the line containing A and B in slope intercept form.

   $$m=\frac{3-(-5)}{-1-3}=\frac{8}{-4}=-2.$$

   Then, using point/slope, $y-3=-2(x+1)$, or $y-3=-2x-2$.

   Hence the equation for this line is: $y=-2x+1$.

   (d) (5 points) Find the equation for the line perpendicular to the line through A and B and containing the point $(-1,2)$

   First, since we are looking for a line that is perpendicular to the line found above, we see that $m=\dfrac{1}{2}$.

   Then, using point/slope and the point $(-1,2)$, $y-2=\frac{1}{2}(x+1)$, or $y-2=\frac{1}{2}x+\frac{1}{2}$.

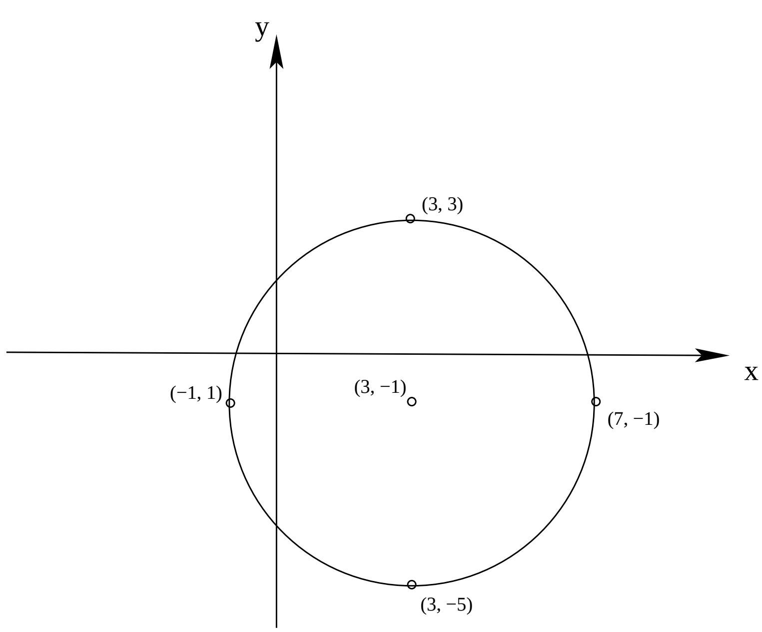
   Hence the equation for this line is: $y=\frac{1}{2}x+\frac{1}{2}+2$ or, simplifying, $y=\frac{1}{2}x+\frac{5}{2}$.

2. (6 points) Graph the circle with equation $x^2+y^2-6x+2y-6=0$

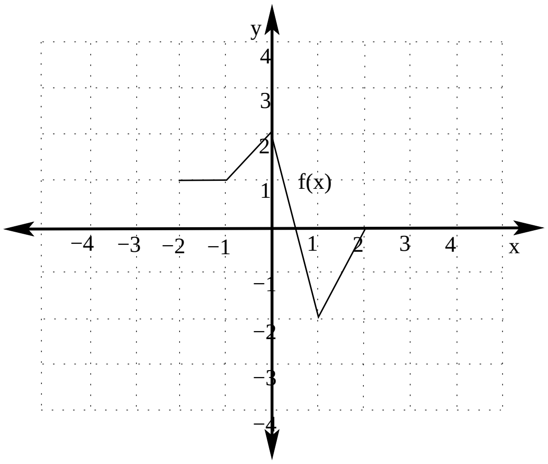
   We begin by completing the square on each quadratic in the equation: $(x^2-6x+9)+(y^2+2y+1)=6+9+1$

   Hence the equation for this circle is: $(x-3)^2+(y+1)^2=16$, so this circle has center $(3,-1)$ and radius $r=4$.

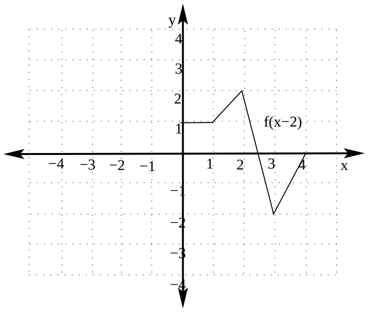
   Therefore, this circle has the following graph.



3. (5 points each) Given the graph of $f(x)$ shown below, use graph transformations to graph each of the following. Label at least 3 points on your final graphs.

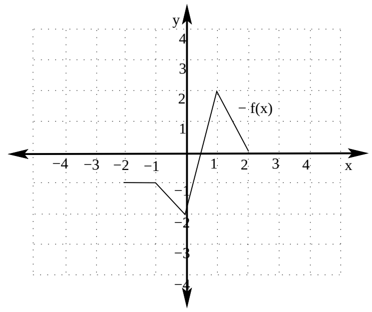


(a) $f(x-2)$



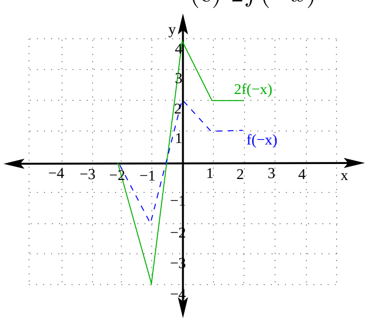
Shift Right 2 units

(b) $-f(x)$



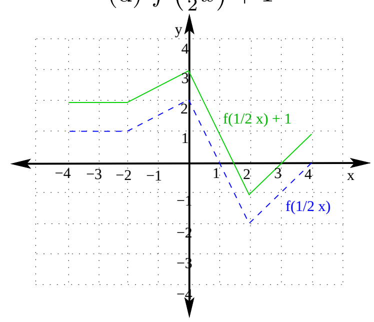
x - axis reflection

(c) $2f(-x)$



x - axis reflection, the vertically stretch by a factor of 2

(d) $f\left(\frac{1}{2}x\right)+1$



Horizontally stretch by a factor of 2, then up one unit.

4. Given that $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{3x-2}$

(a) (3 points) Find $(f \circ g)(x)$

$f(g(x)) = f\left(\sqrt{3x-2}\right)$

$= \frac{1}{\sqrt{3x-2}} = \frac{\sqrt{3x-2}}{3x-2}$

(b) (4 points) Find the domain of $(f \circ g)(x)$

Notice that for g, we must have $3x - 2 \geq 0$ or $3x \geq 0$, and hence $x \geq \frac{2}{3}$

Since the input to $f(x)$ cannot be zero, $x \neq \frac{2}{3}$.
Hence the domain of $(f \circ g)(x)$ is:
$\left(\frac{2}{3}, \infty\right)$.

(c) (3 points) Find $(f \circ g)(2)$

$$f(g(2)) = f\left(\sqrt{4}\right) = f(2) = \frac{1}{2}$$

(d) (3 points) Find $(g \circ f)(1)$

$$g(f(1)) = g(1) = \sqrt{3(1)-2} = \sqrt{1} = 1.$$

5. (3 points each) Given the tables below, find the following:

x	$f(x)$	$g(x)$
0	-4	1
1	2	-3
2	1	4
3	7	2
4	1	0

(a) $(f+g)(3) = f(3)+g(3) = 7+2 = 9$

(b) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{1}{4}$

(c) $(f \circ g)(2) = f(g(2)) = f(4) = 1$

(d) $f^{-1}(2) = 1$ [since $f(1) = 2$]

(e) $f(g^{-1}(0))$

Since $g^{-1}(0) = 4$, then $f(g^{-1}(0)) = f(4) = 1$

6. (7 points) Find the inverse of the function $f(x) = \frac{3x-1}{2x+5}$. You **do not** need to compute compositions to verify your result.

$$y = \frac{3x-1}{2x+5}$$

Multiplying both sides by $2x+5$ gives: $y(2x+5) = 3x - 1$

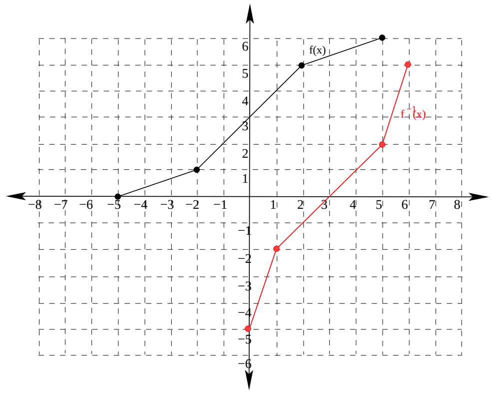
Then $2xy + 5y = 3x - 1$ or $5y + 1 = 3x - 2xy$

Hence $5y + 1 = x(3 - 2y)$

so $x = \frac{5y+1}{3-2y}$

Therefore, $f^{-1}(x) = \frac{5x+1}{3-2x}$

7. Given the following graph of $f(x)$:



(a) (4 points) Find the average rate of change of $f(x)$ in the interval $[-5, 5]$

From the graph above, we see that $f(-5) = 0$, and $f(5) = 6$.

Therefore, the average rate of change of $f(x)$ over this interval is given by $m = \dfrac{6-0}{5-(-5)} = \dfrac{6}{10} = \dfrac{3}{5}$.

(b) (4 points)Draw the graph of $f^{-1}(x)$ on the axes above.

See graph above.

(c) (4 points) Give the domain and range of $f^{-1}(x)$

Notice that the Domain of $f(x)$ is $[-5, 5]$ and the Range of $f(x)$ is $[0, 6]$.

Therefore, the Domain of $f^{-1}(x)$ is $[0, 6]$ and the Range of $f^{-1}(x)$ is $[-5, 5]$.

8. In 1975, U.S. First Class postage cost \$0.10 per stamp. In 1985, the price of stamps had gone up to \$0.22

(a) (4 points) Find a linear function that gives the cost of First Class postage as a function of time in years since 1975.

Notice that x represents time in years since 1975. If we let y be the price of a first class stamp in cents (you could also choose dollars for your units as well, but using cents is a bit nicer), then we have the points $(0, 10)$ and $(10, 22)$.

Then $m = \dfrac{22-10}{10-0} = \dfrac{12}{10} = \dfrac{6}{5} = 1.2$, and the y-intercept is $(0, 10)$.

Then we have the line $y = 1.2x + 10$, so our linear model is $f(x) = 1.2x + 10$.

(b) (3 points) Explain what the slope of your linear function means in practical terms.

Notice that the units of the slope are price in cents per year. Thus the meaning of the slope value of 1.2 is that the price of a first class stamp went up by 1.2 cents per year, on average, each year during the time period from 1975 to 1985.

(c) (4 points) Use your function to predict the cost of a First Class postage stamp today.

First notice that today, 2013, corresponds to $x = 2013 - 1975 = 38$.

Then, using out model, $f(38) = (1.2)(38) + 10 = 45.6 + 10 = 55.6$, so our model predicts that the cost of a first class stamp today is 56 cents (rounding to the nearest cent).