Math 127 **Name:**______________________________

Exam 4 - Version 1

06/18/2013

You MUST show appropriate work to receive credit

1. (5 points each) Solve the following systems of linear equations:

$$\begin{cases} 4x + 3y = -8 \\ 5x + 2y = 11 \end{cases}$$

We multiply the first equation by 2 and the second equation by -3, yielding:

$$\begin{cases} 8x + 6y = -16 \\ -15x - 6y = -33 \end{cases}$$

Adding these equations gives:

$-7x = -49$, or $x = 7$

Substituting this value into the original first equation gives:

$4(7) + 3y = -8$, or $28 + 3y = -8$

Then $3y = -36$, or $y = -12$.

Hence the solution is: $(7, -12)$.

$$\begin{cases} 4x - 6y = 12 \\ 2x - 3y = 6 \end{cases}$$

First, we multiply the second equation by -2:

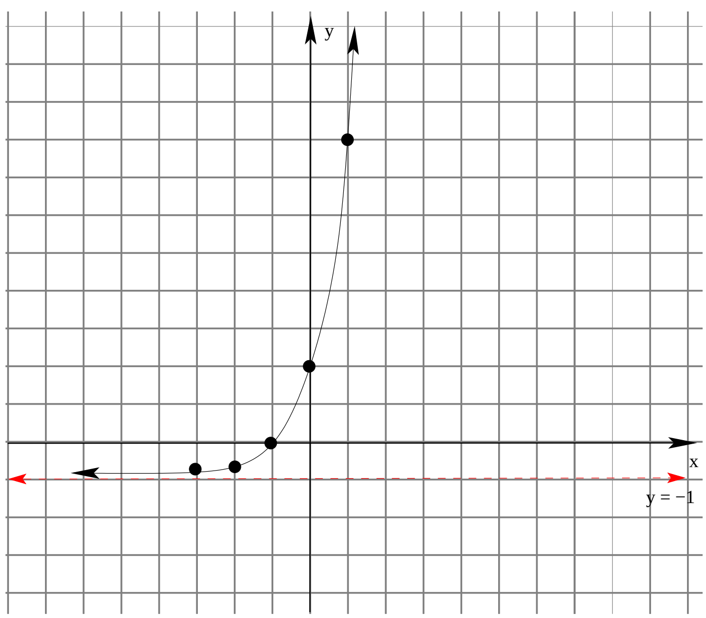
$$\begin{cases} 4x - 6y = 12 \\ -4x + 6y = -12 \end{cases}$$

Adding these gives $0 + 0 = 0$ which is always true.

Hence this system has infinitely many solutions.

That is, every point on the line $2x - 3y = 6$ is a solution to this system.

2. (7 points) Graph the function $f(x) = 3^{x+1} - 1$. Graph and label the asymptote and at least 3 points on your graph. Also give the domain and range.



x	3^{x+1}	$f(x) = 3^{x+1} - 1$
-3	$3^{-2} = \frac{1}{9}$	$-\frac{8}{9}$
-2	$3^{-1} = \frac{1}{3}$	$-\frac{2}{3}$
-1	$3^0 = 1$	0
0	$3^1 = 3$	2
1	$3^2 = 9$	8

Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Asymptote:
the horizontal line $y = -1$

3. (5 points) Suppose you have \$7,000 to invest. Find the amount you would have after 12 years if you deposit your \$7,000 in an account that pays 5% annual interest compounded monthly.

Recall that $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$. Here, $A = \$7000$, $r = 0.05$, $n = 12$, and $t = 12$.

Then we have $A = 7,000\left(1 + \frac{0.05}{12}\right)^{(12)(12)} \approx \$12,738.94$

4. (2 points each) Find the *exact value* of each of the following:

(a) $\log_3(81) = 4$

Since $3^4 = 81$

(b) $\log_{23}(1) = 0$

Since $23^0 = 1$

(c) $\log_\pi(\sqrt[3]{\pi})$ $\log_\pi(\sqrt[3]{\pi}) = \log_\pi\left(\pi^{\frac{1}{3}}\right) = \frac{1}{3}$

(d) $\log_2\left(\frac{1}{8}\right) = -3$

Since $2^{-3} = \frac{1}{8}$

(e) $\log_{25}(5) = \dfrac{1}{2}$

Since $25^{\frac{1}{2}} = 5$

(f) $21^{\log_{21}(7)} = 7$

By the inverse function composition property.

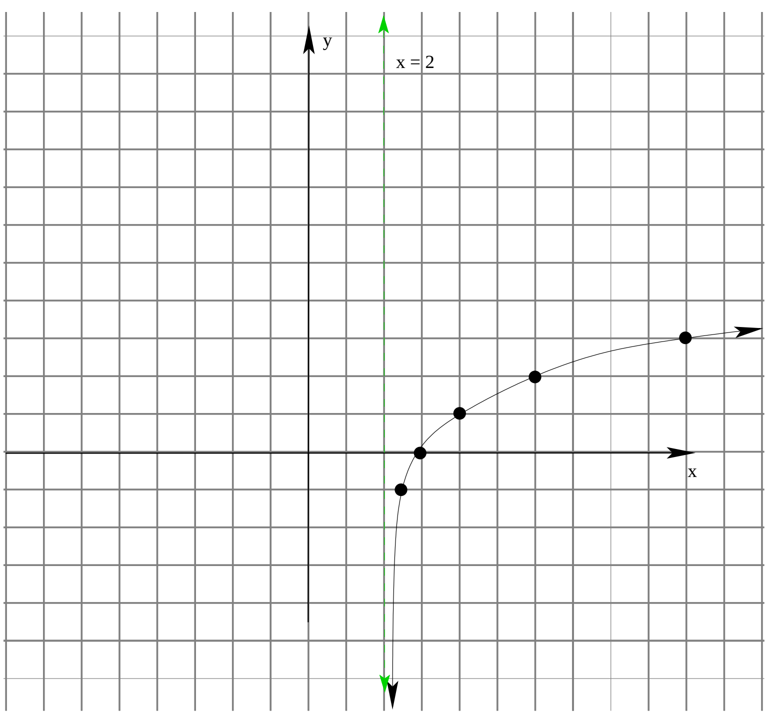
5. (5 points) Use properties of logarithms to **expand** the expression: $\ln\left(\sqrt{\dfrac{x^2y^3}{e^4z^2}}\right)$

$$= \ln\left(\sqrt{\frac{x^2y^3}{e^4z^2}}\right)^{\frac{1}{2}} = \frac{1}{2}\ln\left(\sqrt{\frac{x^2y^3}{e^4z^2}}\right)$$

$$= \frac{1}{2}\ln\left(x^2y^3\right) - \frac{1}{2}\ln(e^4z^2) = \frac{1}{2}\ln\left(x^2\right) + \frac{1}{2}\ln\left(y^3\right) - \frac{1}{2}\ln(e^4) - \frac{1}{2}\ln\left(z^2\right)$$

$$= \ln x + \frac{3}{2}\ln y - 4\cdot\frac{1}{2}\ln e - \ln z = \ln x + \frac{3}{2}\ln y - 2 - \ln z$$

6. (6 points) In grid provided, sketch the graph of the function $g(x) = \log_2(x-2)$, clearly labeling at least 3 points. Also label any asypmtotes, and give the domain and range.



$x-2$	x	$g(x) = \log_2(x-2)$
3	1	0
4	2	1
6	4	2
10	8	3
$\frac{5}{2}$	$\frac{1}{2}$	-1
$\frac{9}{4}$	$\frac{1}{4}$	-2

Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

Asymptote:
the vertical line $x = 2$

7. (5 points) Use the change of base formula to compute $\log_{13}(500)$ to 4 decimal places.

$$\log_{13}(500) = \frac{\ln 500}{\ln 13} \approx 2.4229$$

8. (6 points each) Solve the following equations (give exact answers whenever possible):

(a) $2^{3-2x} = 8^{x+2}$

$2^{3-2x} = \left(2^3\right)^{x+2}$

So $3 - 2x = 3(x+2)$ or $3 - 2x = 3x + 6$.

Then $-3 = 5x$, thus $x = -\frac{3}{5}$

(b) $\log_5(2x - 11) = 3$

Rewriting in exponential form: $5^3 = 2x - 11$, or $125 = 2x - 11$

Then $136 = 2x$, and hence $x = 68$.

Check: (we must check logarithmic equations)

$\log_5(2(68) - 11) = \log_5(136 - 11) = \log_5(125) = 3$ ✓

(c) $5e^{2x} = 15$

We begin by dividing both sides by 5: $e^{2x} = 3$

Then $\ln\left(e^{2x}\right) = \ln(3)$

Then $2x = \ln 3$, so $x = \frac{\ln 3}{2} \approx 0.5493$

(d) $e^{2x} - 2e^x - 3 = 0$

We begin by substituting $u = e^x$: $u^2 - 2u - 3 = 0$

This factors to give $(u-3)(u+1) = 0$, so $u = 3$ or $u = -1$

Then either $e^x = 3$ or $e^x = -1$, so $x = \ln 3$ or $x = \ln -1$.

We see that the first answer works, but the second is impossible
since -1 is outside the domain of $\ln x$.

We retain the second solution: $x = \ln 3 \approx 1.0986$.

(e) $\log(x) + \ln(x - 3) = 1$

This was an unfortunate typo. The problem should have read: $\log(x) + \log(x - 3) = 1$

We begin by combining the left hand side into a single logarithmic expression:

$\log\left(x(x-3)\right) = 1$

Then, rewriting in exponential form: $10^1 = x(x-3)$

Then $10 = x^2 - 3x$ or $x^2 - 3x - 10 = 0$

Factoring this gices $(x-5)(x+2) = 0$

This given two potential solutions: $x = 5$ and $x = -2$.

Check: (we must check logarithmic equations)

Notice that when $x = -2$ $\log(-2)$ and $\log(-2-3) = \log(-5)$ are undefined, so we reject this solution.

When $x = 5$, $\log(5) + \log(5-3) = \log(5) + \log(2) = \log(5 \cdot 2) = \log 10 = 1$ ✓

Thus there is one solution: $x = 5$.

(f) $2^{3x+1} = 5^{2x}$

Taking the logarithm of each side: $\ln 2^{3x+1} = \ln 5^{2x}$

Then $(3x+1)\ln 2 = 2x \cdot \ln 5$, or, distributing, $(3\ln 2)x + \ln 2 = (2\ln 5)x$

Moving terms, we have $\ln 2 = (2\ln 5)x - (3\ln 2)x$

Therefore, $\ln 2 = x(2\ln 5 - 3\ln 2)$

Thus $x = \dfrac{\ln 2}{2\ln 5 - 3\ln 2} \approx 0.60833$

9. Suppose that the population of Algebronia is growing according to the model $f(t) = 15e^{0.043t}$ where t is in years since 2000, and $f(t)$ is in millions of people.

   (a) (3 points) Find the population of Algebronia in the year 2000.

   Notice that the year 2000 corresponds to $t = 0$. so the population in 2000 is given by $f(0) = 15e^{0.043(0)} = 15e^0 = 15$, or 15 million people.

   Therefore, there were 15 million people living in Algebronia in the year 2000.

   (b) (3 points) Find the populations of Algebronia today (to the nearest person).

   Notice that today corresponds to $t = 2013 - 2000 = 13$, so the population today is given by $f(13) = 15e^{0.043(13)} \approx 26.233841$

   So there are 26,233,841 people living in Algebronia today (notice that you were asked to find the population to the *nearest person*).

   (c) (5 points) Find the year that the population of Algebronia reaches 50 million people.

   To find the year in which the population will reach 50 million, we solve the equation $50 = 15e^{0.043t}$

   Then $\frac{50}{15} = e^{0.043t}$, or $\ln\left(\frac{50}{15}\right) = 0.043t$

   Hence $t = \dfrac{\ln\left(\frac{50}{15}\right)}{0.043} \approx 27.999$, or in the year 2028.

10. (7 points) Suppose that laboratory research has shown that a 10 gram sample of a newly discovered substance called Strawberronium is reduced down to 7 grams in 6 hours. Find the half life of Strawberronium.

    We will use the continuous decay model: $A = A_0e^{kt}$. To find the half life, we must first use the information we know about Strawberrionium to find the growth constant k. In the lab test, we had $A_0 = 10$ grams, $A = 7$ grams, and $t = 6$ hours.

    Then $7 = 10e^{6k}$, so $\frac{7}{10} = e^{6k}$, or $\ln\frac{7}{10} = 6k$. Thus $k = \frac{\ln\frac{7}{10}}{6} \approx -0.059446$.

    Using this, we set up the half-life equation: $\frac{1}{2} \approx e^{-0.059446t}$.

    Solving this for t, we have $\ln\frac{1}{2} \approx -0.059446t$, so $t \approx \frac{\ln\frac{1}{2}}{-0.059446} \approx 11.66$

    Hence the half-life of Strawberronium is about 11.7 hours.

Extra Credit: (5 points) Solve the Equation $\log_2(\log_3(\log_4 x)) = 1$

First, re-writing in exponential form, $2^1 = \log_3(\log_4 x)$, or $2 = \log_3(\log_4 x)$

Rewriting again, we have $3^2 = \log_4 x$, or $9 = \log_4 x$.

Finally, rewriting a third time, $4^9 = x$, hence $x = 262,144$.