Math 261 – Lab 17 **Capstone B** Name_________________

Show all work for credit. Give exact answers unless otherwise noted.

1. *(From the 2006 AP Calculus exam.)* A rocket has positive velocity, $v(t)$, after being launched upward from an initial height of zero feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected times over the interval from 0 to 80 seconds, as shown in the table below.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

(a) Find the average acceleration of the rocket over the 80 second time interval. Indicate the units of measure.

(b) Using correct units, explain the meaning of $\int_{10}^{70} v(t)\,dt$ in terms of the rocket's flight.

(c) Use Simpson's rule with the six subintervals indicated by the table to approximate $\int_{10}^{70} v(t)\,dt$.

(d) A second rocket is launched upward with an acceleration of $a(t) = \dfrac{3}{\sqrt{t+1}}$ feet per second per second.

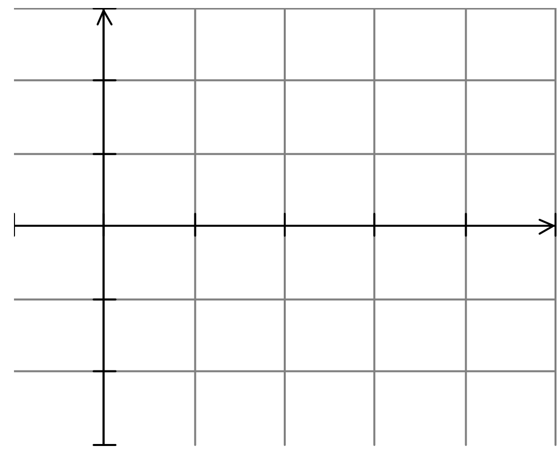
At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

2. *(From the 2005 AP Calculus AB exam.)* Let f be a function that is continuous on the interval [0, 4). The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table below. (DNE represents *does not exist*.)

x	0	(0, 1)	1	(1, 2)	2	(2, 3)	3	(3, 4)
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify.

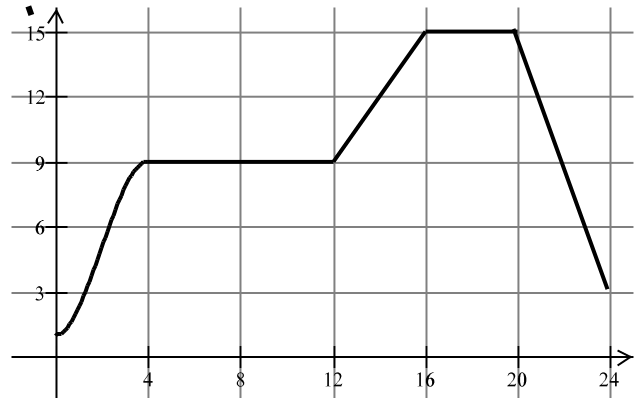
(b) On the coordinate plane below, sketch the graph of a function that has all the characteristics of f.



(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval (0, 4). For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of $x \in (0, 4)$ at which the graph of g has a point of inflection. Justify your answer.

3. *(Adapted from the 2006 AP Calculus AB exam.)* The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function C. In the figure below, $C(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $t \in [0, 4]$ and C is piecewise linear for $t \in [4, 24]$.



(a) Find each of the following limits, if possible.

(i) $\lim_{t \to 2} C(t)$ (ii) $\lim_{t \to 4} C(t)$

(iii) $\lim_{t \to 8} C(t)$ (iv) $\lim_{t \to 12} C(t)$

(v) $\lim_{t \to 22} C(t)$ (vi) $\lim_{t \to 24^-} C(t)$

(vii) $\lim_{t \to 0^+} C'(t)$ (viii) $\lim_{t \to 2} C'(t)$ (ix) $\lim_{t \to 4^-} C'(t)$ (x) $\lim_{t \to 4^+} C'(t)$

(xi) $\lim_{t \to 4} C'(t)$ (xii) $\lim_{t \to 12^-} C'(t)$ (xiii) $\lim_{t \to 12^+} C'(t)$ (xiv) $\lim_{t \to 12} C'(t)$

(xv) $\lim_{t \to 14} C'(t)$ (xvi) $\lim_{t \to 18} C'(t)$ (xvii) $\lim_{t \to 22} C'(t)$ (xviii) $\lim_{t \to 24^-} C'(t)$

(xix) $\lim_{t \to 24} C'(t)$ (xx) $\lim_{t \to 14} C''(t)$ (xxi) $\lim_{t \to 12} C''(t)$ (xxii) $\lim_{t \to 2} C''(t)$

(b) Find any time values in (0, 24) where C is not continuous.

(c) Find any time values in (0, 24) where C' is not continuous.

(d) At what time is the rate at which the person is burning calories, C, increasing the greatest? Show the reasoning that supports your answer.

(e) Find the total number of calories burned over the time interval from 6 to 18 minutes.

(f) The setting on the machine is now changed so that the person burns $C(t) + k$ calories per minute. For this setting, find k so that an average of 15 calories per minute is burned during the time interval from 6 to 18 minutes.