1. Express the following numbers in scientific notation:

   (a) \( 0.00211 = 2.11 \times 10^{-4} \)

   (b) \( -0.00945 = -9.45 \times 10^{-3} \)

   (c) \( 23,456,000 = 2.3456 \times 10^7 \)

   (d) \( -3,670,000 = -3.67 \times 10^6 \)

2. Express the following numbers in ordinary decimal notation:

   (a) \( 5.67 \times 10^{-5} = 0.0000567 \)

   (b) \( -4.02 \times 10^{-3} = -0.00402 \)

   (c) \( 1.45 \times 10^3 = 1,450 \)

   (d) \( -9.99 \times 10^4 = -99,900 \)

3. Say if each of the following statements is true or false. If false give a counter-example. The parameters \( a, b, \) and \( c \) are real numbers (NOT NECESSARY POSITIVE) and we assume that all denominators are different from 0.

   (a) \( \frac{ab + c}{a} = b + c \) \quad False \quad Choose \( a = 2, b = 1, c = 4 \)

   \[ \frac{2 \cdot 1 + 4}{2} = 3 \neq 1 + 4 = 5 \]

   (b) \( \frac{a + c}{b + d} = \frac{a}{b} + \frac{c}{d} \) \quad False \quad Choose \( a = b = c = d = 1 \)

   \[ \frac{1 + 1}{1 + 1} = \frac{2}{2} = 1 \neq \frac{1}{1} + \frac{1}{1} = 2 \]

   (c) \( \frac{a}{b + c} = \frac{a}{b} + \frac{a}{c} \) \quad False \quad Choose \( a = 4, b = c = 2 \)

   \[ \frac{4}{2 + 2} = \frac{4}{4} = 1 \neq \frac{4}{2} + \frac{4}{2} = 2 + 2 = 4 \]

   (d) \( \sqrt{a^2 + b^2} = a + b \) \quad False \quad Choose \( a = 3, b = 4 \)

   \[ \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \neq 3 + 4 = 7 \]
(e) $\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b}$ True

(f) $\frac{a - b}{b - a} = -1$ True

(g) $\sqrt{a^2} = a$ False Choose $a = -1$
$\sqrt{(-1)^2} = \sqrt{1} = 1 \neq -1$

(h) $(a + b)^2 = a^2 + b^2$ False Choose $a = b = 1$
$(1 + 1)^2 = 4 \neq 1 + 1 + 1 = 2$

(i) $(a + b)^2 = a^2 + 2ab + b^2$ True

4. Evaluate each of the following

(a) $(-3)^2 = 9$  
(c) $\sqrt{-27} = -3$  
(e) $\sqrt{-4}$ undefined

(b) $-3^2 = -9$  
(d) $\sqrt{50} = \frac{5}{2}$  
(f) $(-2)^{\frac{1}{4}}$ undefined

5. Simplify each of the following. Assume that any variables are positive. Your answer should have all radicals reduced entirely, denominators rationalized, and have no negative exponents (3 points each).

(a) $\sqrt[3]{108xy^4 - y\sqrt[3]{32xy}} = 3y\sqrt[4]{4x} - 2y\sqrt[4]{4xy} = -y\sqrt[4]{4xy}$
(b) $\frac{(x^2y^3)^4(3xy^4)^{-3}}{x^2y} = 3^{-3,2}x^{2-4}y^{3-4+4(-3)-1} = x^3 \frac{27y}{x^3} = 27y$
(c) $\left(\frac{x^3}{a^6}\right)^{-\frac{3}{2}} = \left(\frac{4x^3a}{x^3a^6}\right)^{-\frac{3}{2}} = 2x^\frac{3}{2}a^\frac{1}{2} = 2\sqrt{x}\sqrt[4]{a}$
(d) $\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$
(e) $\frac{x - 25}{\sqrt{x} - 5} = \frac{x - 25}{\sqrt{x} - 5} \cdot \frac{\sqrt{x} + 5}{\sqrt{x} + 5} = \frac{(x - 25)(\sqrt{x} + 5)}{x - 25} = \sqrt{x} + 5$
(f) $\left(\frac{(5xyz)^2z^{-2}}{2x^{-2}y^2z^{-4}}\right)^{-1} = \frac{(5xyz)^{-2}z^2}{2^{-1}x^2y^{-2}z^4} = \frac{5^{-2}x^{-2}y^{-2}z^{-2} \cdot z^2}{2^{-1}x^2y^{-2}z^4} = \frac{2}{25x^4z^4}$
6. Perform the indicated operations and simplify

(a) \[3(2x^3 - x^2 + 5x) - 2x(3x^3 - 2x^2 + 5x - 3) = \]
\[= 6x^3 - 3x^2 + 15x - 6x^4 + 4x^3 - 10x^2 + 6x = \]
\[= \frac{-6x^4 + 10x^3 - 13x^2 + 21x}{x - 1}\]
(b) \[(2x^2 + 3x - 2)(x - 2) = \]
\[= 2x^3 - 4x^2 + 3x^2 - 6x - 2x + 4 = \]
\[= \frac{2x^3 - x^2 - 8x + 4}{x - 1}\]
(c) \[(2x - 1)^3 = \]
\[= (2x)^3 - 3(2x)(1) + 3(2x)(1)^2 - (1)^3 = \]
\[= 8x^3 - 12x^2 + 6x - 1\]
(d) \[\left(\frac{x^\frac{1}{3} - y^\frac{1}{3}}{x^\frac{2}{3} + x^\frac{1}{3}y^\frac{1}{3} + y^\frac{2}{3}}\right) = \]
\[= x + x^\frac{2}{3}y^\frac{1}{3} + x^\frac{1}{3}y^\frac{2}{3} - x^\frac{2}{3}y^\frac{1}{3} - x^\frac{1}{3}y^\frac{2}{3} - y = \]
\[= \frac{x - y}{x - 1}\]

7. Factor each of the following expressions completely:

(a) \[2x^2 + x - 6 = 2x^2 + 4x - 3x - 6 = 2x(x + 2) - 3(x + 2) = \frac{(2x - 3)(x + 2)}{x - 1}\]
(b) \[50x^2 + 45x - 18 = 50x^2 + 60x - 15x - 18 = 10x(5x + 6) - 3(5x + 6) = \frac{(10x - 3)(5x + 6)}{x - 1}\]
(c) \[9x^2 - 49y^6 = (3x)^2 - (7y^3)^2 = \frac{(3x - 7y^3)(3x + 7y^3)}{x - 1}\]
(d) \[8x^3 - y^3 = (2x)^3 - (y)^3 = \frac{(2x - y)(4x^2 + 2xy + y^2)}{x - 1}\]
(e) \[6x^3y - 27x^2y - 15xy = 3xy(2x^2 - 9x - 5) = 3xy(2x + x - 10x - 5) = \frac{3xy[x(2x + 1) - 5(2x + 1)]}{x - 1}\]
\[= \frac{3xy(2x + 1)(-5)}{x - 1}\]
(f) \[3x^3 + x^2 - 3x - 1 = x^2(3x + 1) - (3x + 1) = (x^2 - 1)(3x + 1) = \frac{(x - 1)(x + 1)(3x + 1)}{x - 1}\]
(g) \[x^6 - 1 = (x^3 - 1)^2 = (x^3 - 1)(x^3 + 1) = \frac{(x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)}{x - 1}\]

8. In the following perform the indicated operations. Your final answer must be a single simplified fraction.

(a) \[\frac{3x^2 - 10x + 3}{x^2 - 1} \cdot \frac{x^2 + x - 2}{x^2 - 9} = \frac{(3x - 1)(x - 3)}{(x + 1)(x - 1)} \cdot \frac{(x + 2)(x - 1)}{(x + 3)(x - 3)} = \frac{(3x - 1)(x + 2)}{(x + 1)(x + 3)}\]
(b) \[\frac{x^2 - 4}{x - 2} \div \frac{x + 2}{4x - 8} = \frac{(x - 2)(x + 2)}{(x - 2)} \cdot \frac{4(x - 2)}{x + 2} = \frac{4(x - 2)}{x + 2}\]
(c) \[\frac{4}{x} + \frac{3}{x + 3} = \frac{4(x + 3) + 3x}{x(x + 3)} = \frac{7x + 12}{x(x + 3)}\]
(d) \[\frac{2x^2 + 4}{2x^2 + 7x - 4} - \frac{x - 1}{x + 4} = \frac{2x^2 + 4}{(2x - 1)(x + 4)} - \frac{x - 1}{x + 4} = \frac{2x^2 + 4 - (x - 1)(2x - 1)}{(2x - 1)(x + 4)} = \frac{3(x + 1)}{(2x - 1)(x + 4)}\]
9. In the following simplify each complex rational expression.

(a) \[ \frac{\frac{1}{x} + \frac{3}{x-2}}{\frac{2}{x-2}} = \frac{\frac{x-2+3x}{x(x-2)}}{\frac{2}{x-2}} = \frac{\frac{4x-2}{x(x-2)}}{\frac{2x-6}{(x-1)(x-2)}} = \frac{x(x-1)(x-2)}{(x-1)(x-2)} = \frac{(4x-2)(x-1)}{(2x-6)x} \]

(b) \[ \frac{\frac{1}{2x+2h+1} - \frac{1}{2x+1}}{h} = \frac{\frac{2x+1-2x-2h-1}{(2x+2h+1)(2x+1)}}{h} = -\frac{\frac{2h}{(2x+2h+1)(2x+1)}}{h} = \frac{2}{(2x+2h+1)(2x+1)} \]

10. Suppose a square garden has an area represented by \(9x^2\) square feet. If one side is made 7 feet longer and the other side is made 2 feet shorter, write the trinomial modeling the area of the new garden.

**Sol:**

the side of the square is \(3x\), if we make it 7 feet longer we get \(3x + 7\), if we make it 2 feet shorter we get \(3x - 2\), therefore the area of the new garden is \((3x + 7)(3x - 2) = 9x^2 - 6x + 21x - 14 = 9x^2 + 15x - 14\)

11. The price \(x\) on a plasma television had been reduced by 20%, then it has been reduced of another 15%. If taxes for this item are 7%, write a monomial representing how much you will pay for the plasma television when you check out.

**Sol:**

When you check out you will pay \(1.07 \times 0.85 \times 0.80 \times x = \$0.7276x\).

12. The volume of a cube whose side is \(2x + 1\), equals the area of a square. Write a (factored) algebraic expression representing the side of the square.

**Sol:**

The volume of the cube is \((2x + 1)^3\), if this is also the area of a square, the side of the square is \(\sqrt{(2x + 1)^3} = (2x + 1)^{3/2}\).