Ch. 10 Gases and the Ideal Gas Law(s)

10.1 The Atmosphere
1. Earth surrounded by gas
2. Major components:
   • Nitrogen 78%
   • Oxygen 21%
   • Miscellaneous: All <1%
   • Argon, carbon dioxide, neon, hydrogen, helium, methane, ozone, etc.

10.2 Properties of Gases

General Properties
1. Can be compressed
2. Exert pressure
3. Expand into whatever volume is available
4. Mix completely

Four Defining Variables for Any Pure Gas
1. Amount
   • Moles is what matters in most calculations
   • Interconversions from grams to moles, or from moles to grams, is involved in many problems
2. Volume
3. Pressure
4. Temperature
   • In Kelvin

STP: Standard Temperature and Pressure
• 0º C (273 K)
• 1 atmosphere (760 mm Hg)
• 1 mole == 22.4 L for an ideal gas at STP

Several Key Conversion Factors When Dealing with Gases

<table>
<thead>
<tr>
<th>Conversion Factors</th>
<th>Interconversion Between:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 1.0 atm = 760 mm Hg</td>
<td>Pressure units</td>
</tr>
<tr>
<td>2. K = 273 + ºC</td>
<td>Temperature units</td>
</tr>
<tr>
<td>3. ( x ) moles = ( \frac{\text{actual mass}}{\text{known molar mass}} )</td>
<td>Mass/mole interconversions</td>
</tr>
<tr>
<td>4. 1 mole = 22.4 L at STP</td>
<td>mole/volume interconversions at STP</td>
</tr>
</tbody>
</table>
10.3 Kinetic-Molecular Theory
1. Gas molecules are far apart
   - mostly empty space
2. Gas molecules move randomly at various speeds and in every possible direction
3. Except when molecules collide, forces of attraction and repulsion between them are negligible.
   - IMF don’t attract them together, unlike with liquids or solids
4. Collisions occur at times and are “elastic”
   - A collision does not impact the combined energy.
   - But the directions and speeds of the colliding gases may change.
5. The average kinetic energy of gas molecules is proportional to absolute temperature (in Kelvin)

Practical Temperature-Related Notes:
1. Not all molecules at a given temperature have the same energy.
   - An increase in temperature significantly increases the population of high energy molecule
2. An increase in temperature significantly increases the population of high energy molecule
3. Smaller molecules move much faster (on average) than larger molecules at a given temp
   - They need much more speed to have the same average energy
   - Average speed is not directly related to temperature \( (E = mv^2) \)

Practical Pressure: Movement \( \rightarrow \) collisions with walls/surface \( \rightarrow \) PRESSURE
1. More hits \( \rightarrow \) more pressure
   - Higher temp \( \rightarrow \) faster average speed \( \rightarrow \) more hits \( \rightarrow \) more pressure
   - More volume \( \rightarrow \) fewer molecules per area \( \rightarrow \) fewer hits \( \rightarrow \) less pressure
   - Less volume \( \rightarrow \) more molecules per area \( \rightarrow \) more hits \( \rightarrow \) higher pressure
   - More gas (more moles) \( \rightarrow \) more molecules per area \( \rightarrow \) more hits \( \rightarrow \) higher pressure
2. More energetic hits \( \rightarrow \) more pressure
   - Higher temp \( \rightarrow \) faster average speed \( \rightarrow \) harder hits \( \rightarrow \) more pressure

\( \text{Higher temp } \rightarrow \text{higher pressure} \)
\( \text{Smaller volume } \rightarrow \text{higher pressure} \)

Small Gases “escape” through tiny holes faster than big molecules
- Helium balloons vs. air balloons

10.4-7 Gas “Laws”
“ideal” gases all behave the same: same properties, laws

<table>
<thead>
<tr>
<th>The Ideal Gas Law</th>
<th>PV = nRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rearranged Versions</td>
<td>( V = \frac{nRT}{P} )</td>
</tr>
</tbody>
</table>

\( P = \) Pressure (atm)
\( V = \) Volume (L)
\( n = \) moles
\( R = \text{gas constant } = 0.0821 \text{ atm}\cdot\text{L/mol}\cdot\text{K} \)
\( T = \text{Temperature (Kelvin)} \)

More gas \( \rightarrow \) more volume (if pressure is constant) or more pressure (if volume is fixed)
Higher temperature \( \rightarrow \) more volume (if pressure is constant) or more pressure (if volume is fixed)
More pressure \( \rightarrow \) less volume
More volume \( \rightarrow \) less pressure
Some Problem Types:
  a. Given any 3 of the 4 variables, solve for the 4th.
  b. Given initial conditions, how would changing any variable change others?
  c. STP conditions: 22.4 L = 1 mol

Volume and Pressure (Boyle's Law) \( V \propto \frac{1}{P} \)

1. A balloon at 2.3 atm has a volume of 28 L. What will the volume be at 1.0 atm? (Assume constant temperature)
   \[
   V_1 P_1 = V_2 P_2 \\
   V_2 = \frac{28 \text{ L} \cdot (2.3 \text{ atm})}{1 \text{ atm}} = 64.4 \text{ L}
   \]

Volume and Pressure (Boyle's Law) \( V \propto \frac{1}{P} \)

2. A gas at 740 mm Hg has volume 720 L. What pressure in atm is needed to reduce the volume to 175 L? (Assume constant temperature)

   Key: Convert pressure to atm
   \[
   x \text{ atm} = \frac{740 \text{ mm Hg}}{760 \text{ mm Hg}} = 0.974 \text{ atm}
   \]
   \[
   P_f = \frac{720 \text{ L} \cdot 1 \text{ atm}}{175 \text{ L}} = 4.01 \text{ atm}
   \]

3. The volume of a gas increases from 3.0 to 9.0 L. If the original pressure was 3 atm, what is the final pressure?

   Volume increases 3-fold
   Pressure decreases 3-fold
   \[
   3 \text{ atm} \rightarrow 1 \text{ atm}
   \]

Volume and Temperature (Charles's Law) \( V \propto T \) (in Kelvin)

4. What is the volume if 78.0 L of gas is heated from 20°C \( \rightarrow 100°C \)? (Assume constant pressure)

   Key: Always convert °C to Kelvin
   \[
   20°C = 293 \text{ K} \\
   100°C = 373 \text{ K}
   \]
   \[
   V_f = \frac{780 \text{ L} \cdot 373 \text{ K}}{293 \text{ K}} = 99.3 \text{ L}
   \]
Volume and Moles (Avogadro’s Law) V ∝ n (in moles)
5. A container with 1.0 mole of CO₂(g) has a volume of 22.4 L. What will be the volume if:
   a. 2 more moles is added
      \[ 22.4 \times \frac{3 \text{ mol}}{1 \text{ mol}} = 67.2 \text{ L} \]
   b. 1 mole of N₂(g) is added?
      \[ 22.4 \times \frac{2 \text{ mol}}{1 \text{ mol}} = 44.8 \text{ L} \]
   c. 0.5 moles escapes through a leak?
      \[ 22.4 \times \frac{0.5 \text{ mol}}{1 \text{ mol}} = 11.2 \text{ L} \]
6. The volume of a gas at STP is 22.4 L/mol. What would be the volume of 12g of CO₂(g) (mw = 44g/mol) at STP?
   \[ \text{Key: } \text{Always convert grams to moles} \]
   \[ \times \frac{1 \text{ mol}}{22.4 \text{ L}} = 0.541 \text{ L} \]
   \[ \text{Key: } \text{At STP, can go from L to moles} \]
   \[ \text{Or from moles to L} \]
   \[ \text{Logic: } \text{g} \rightarrow \text{mol} \rightarrow \text{Liter} \]
7. Would you expect 12g of CH₄(g) (mw = 16g/mol) at STP to have a larger or a smaller volume?
   \[ \text{Larger, } 12g \text{ CH}_4 = 0.75 \text{ mol} \]
   \[ 12g \text{ CO}_2 = 0.27 \text{ mol} \]
   \[ \text{Almost } 3 \text{ times the volume is } \frac{44g}{16g} \]
   \[ \text{The Ideal Gas Law: } PV = nRT \ (R = 0.0821) \]
8. What is the volume for 12 g of N₂ at 1.2 atm and 25°C?
   \[ \text{Key:} \]
   \[ \text{a. Always convert to proper units} \]
   \[ \text{b. Rearrange equation as needed} \]
   \[ 25°C + 273 = 298 K \]
   \[ 12g \text{ N}_2 \times \frac{1 \text{ mol}}{28g} = 0.429 \text{ mol} \]
   \[ \frac{\rho V = nRT}{P} = \frac{(0.429)(1.021)(298)}{1.2} = 8.74 \text{ L} \]
9. How many moles are in a 4.0 L sample of gas at 600 mm Hg and 25°C?
   \[ \text{No, } \times \text{ atm} = \frac{600 \text{ mm Hg}}{1 \text{ atm}} = 0.789 \text{ atm} \]
   \[ n = \frac{\rho V}{RT} = \frac{(4.0)(0.789)}{(1.021)(298)} = 0.13 \text{ mol} \]
10. What is the pressure of 14g Ar(g) (39.9 g/mol) at 52° C in a 4.6 L container?

\[ \rho = \frac{nRT}{V} = \frac{\left( \frac{14g}{39.9g/mol} \right) \cdot 0.0821 \cdot (52 + 273)}{4.6 \text{ L}} = 2.0 \text{ atm} \]

STP: Standard Temperature and Pressure: 0° C (273 K), 1.00 atm. (memorize)

11. Calculate the volume of one mole of gas (any gas) at STP using the ideal gas law.

\[ V = \frac{nRT}{\rho} = \frac{\left( \frac{1}{22.4} \right) \cdot 0.0821 \cdot 273}{1} = 22.4 \frac{L}{1 \text{ mol}} \]

Key: At STP, all gases have the same volume per mole: 22.4 L/1 mole

12. What is the mass of 12 L of N₂ (28 g/mol) at STP?

Key: At STP, can go from L to moles
Or from moles to L

\[ x \rho = \frac{12L}{22.4L/1 \text{ mol}} \cdot 28g/1 \text{ mol} = 15g \]

13. What is the volume of 16 g of O₂ (32 g/mol) at STP?

\[ \rho \rightarrow \text{mol} \rightarrow \text{L} \]

\[ x \rho = \frac{16g}{32g/1 \text{ mol}} \cdot 22.4L/1 \text{ mol} = 11.2L \]

14. What is the density (in g/L) of Ar (40 g/mol) at STP?

\[ \rho = \frac{\rho V}{L} \]

Set volume = \_ L, and solve for moles and grams

\[ x \rho = \frac{\rho 1 \text{ mol}}{22.4L/1 \text{ mol}} \cdot 40g = 1.79 \text{ g/L} \]

[ density = \frac{1.79 \text{ g}}{1 \text{ L}} ]
Gases and Stoichiometry

1. We can easily interconvert between volume and moles
   - Easiest at STP
   - Still possible under any temperature/pressure conditions.

2. Since we can easily use molar mass to interconvert between mass and moles, we can indirectly interconvert between volume and grams.
   - Easiest at STP
   - Still possible at any temperature/pressure conditions.

<table>
<thead>
<tr>
<th>Volume (Liters) ↔ moles ↔ grams</th>
<th>Grams ↔ moles ↔ Volume (liters)</th>
</tr>
</thead>
</table>

3. For different gases under equal temperature/pressure conditions, volume ratios = mole ratios.
   - If given volume ratios in a reaction, you can easily deduce stoichiometry ratios (balanced equations)
   - If given a balanced equation, you can easily deduce volume ratios

15. Balance the reaction by filling in the coefficients, given the following volume information:
    10 L A reacts with 20 L B to give 10 L of C.

    \[ \frac{1}{A} + \frac{2}{B} \rightarrow \frac{1}{C} \]
    \[ 10 \text{ L} \quad 20 \text{ L} \quad 10 \text{ L} \]

16. If 8.0 L of A reacts, how many L of B, C, and D will be consumed or produced, given the following balanced reaction:
    \[ 1A + 2B \rightarrow 2C + 1D \]

    \[ x: B \quad 16 \text{ L} \quad x: C \quad 16 \text{ L} \quad x: D \quad 8 \text{ L} \]

17. How many liters of CO₂ are produced from combustion of 28g of C₆H₈ (56 g/mol) at STP, given the following balanced reaction:
    \[ 1 \text{ C}_6\text{H}_8 + 6 \text{ O}_2 \rightarrow 4 \text{ CO}_2 + 4 \text{ H}_2\text{O} \]

    \[ \frac{\times L}{\text{ CO}_2} = \frac{28 \text{ g} \text{ A}}{56 \text{ g} \text{ A}} \times \frac{1 \text{ mol} \text{ A}}{1 \text{ mol} \text{ CO}_2} \times \frac{22.4 \text{ L} \text{ CO}_2}{1 \text{ mol} \text{ CO}_2} = 44.8 \text{ L} \text{ CO}_2 \]
Review Problems for Simple Conceptual Gas Concepts. Mathematical relationships may exist in some cases, but if so I’m just looking for relative rankings.

18. If 70 students take a test, and the average is 75%, will any students get >90%? Yes

19. If 70 students take a test, and the average is 85%, will any students get >90%? Yes, More

20. Will more students get >90% if the average was 75% or 85%? O

21. Water boils at 100°C. Is it possible for some water molecules to evaporate at 50°C? Why? Yes, Some have above average energy, enough to escape

22. Will more water evaporate faster at 80°C? Why? Yes, More have escape energy, exceed the energy threshold.

23. At 25°C, how does the average kinetic energy compare for N₂ (28 g/mol), O₂ (32 g/mol), and He (4 g/mol)? All same.

24. At 25°C, how does the average speed compare for N₂ (28 g/mol), O₂ (32 g/mol), and He (4 g/mol)?

   He > N₂ > O₂  Smaller are faster.

25. If the walls of a balloon have some tiny pores, what would you expect for the average escape rate (“rate of effusion”) for N₂ (28 g/mol), O₂ (32 g/mol), and He (4 g/mol)?

   He > N₂ > O₂  Smaller escape more easily.

26. At 25°C and 1 atm pressure, how does the volume compare for one mole each of N₂ (28 g/mol), O₂ (32 g/mol), and He (4 g/mol)?

   All same.

27. At 25°C and 1 atm pressure, how does the mass compare for one liter each of N₂ (28 g/mol), O₂ (32 g/mol), and He (4 g/mol)?

   O₂ > N₂ > He  More mass per mole, more mass per L.

28. At 25°C and 1 atm pressure, how does the density compare for N₂ (28 g/mol), O₂ (32 g/mol), and He (4 g/mol)?

   O₂ > N₂ > He.
Key Gas Math Summary

STP: Standard Temperature and Pressure
- 0°C (273 K)
- 1 atmosphere (760 mm Hg)
- 1 mole == 22.4 L for an ideal gas at STP

Several Key Conversion Factors When Dealing with Gases

Conversion Factors
1. 1.0 atm = 760 mm Hg
2. \( K = 273 + ^oC \)
3. \( x \text{ moles} = \frac{\text{actual mass}}{\text{known molar mass}} \)
4. 1 mole = 22.4 L at STP

<table>
<thead>
<tr>
<th>The Ideal Gas Law</th>
<th>( PV = nRT )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rearranged Versions</td>
<td>( V = \frac{nRT}{P} )</td>
</tr>
</tbody>
</table>

\( P = \) Pressure (atm)
\( V = \) Volume (L)
\( n = \) moles
\( R = \) gas constant = 0.0821 atm·L/mol·K
\( T = \) Temperature (Kelvin)

The correct units are essential. Be sure to convert whatever units you start with into the appropriate units when using the ideal gas law.

\( R = 0.0821 \text{ atm·L/mol·K} \)

Density = \( g/L \)

Basic Gas Laws:
- \( V \propto \frac{1}{P} \) or \( PV = \) constant  Boyle’s Law
- \( V \propto T \) or \( \frac{V}{T} = \) constant  Charles’s Law
- \( V \propto n \) or \( \frac{V}{n} = \) constant  Avogadro’s Law
- \( V \propto \frac{T}{P} \) or \( \frac{PV}{T} = \) constant  Combined Law