Ch. 17 Additional Aqueous Equilibria

- Chapter 16 situations basically only involved one solute: strong or weak acid; strong or weak base; or ionic salt
- Real solutions often have >1 solute
  ⇒ more complex equilibria, pH calculations
  ⇒ extra solute impacts degree to which first chemical dissociates

3 general multi-solute situations:
1. buffers: weak acid and weak base mixture, to which strong acid or base may be added
2. titration: mixing acid and base, at least one of which is normally strong
3. partially soluble solids: impact of pH or other salts on solubility

17.1 Buffer solutions: resist change in pH when either acid or base added
- Crucial in nature: blood pH, lake pH, etc. must be tightly controlled.
- Blood: pH ≈ 7.40 ± 0.05
  o pH < 7.35 → acidosis
  o pH > 7.45 → alkalosis
- The body needs a system for keeping the blood at the target pH, and protecting it from the extreme pH disruption that could result from addition of acid or base
- Many cellular processes likewise depend on precise control of pH
- Buffers provide protection against the addition of H⁺ or HO⁻

1. Unbuffered: Find pH
   a. 1.0 L pure water
   b. 1.0 L water + 0.1 mol HCl
   c. 1.0 L water + 0.1 mol NaOH

Point: In an unbuffered, unprotected solution, a small addition of strong acid or base can cause a massive and dangerous shift in pH.

A. Reaction of strong acid (base) with weak base (acid). Background Facts:
While it reacts only slightly with water, a weak acid reacts almost completely with strong base
While it reacts only slightly with water, a weak base reacts almost completely with strong acid

\[
\begin{align*}
\text{NH}_3 + \text{H}_2\text{O} & \rightleftharpoons \text{NH}_4^+ + \text{HO}^- \\
\text{NH}_3 + \text{HCl} & \rightleftharpoons \text{NH}_4^+ + \text{Cl}^- \\
\text{HF} + \text{H}_2\text{O} & \rightleftharpoons \text{H}_3\text{O}^+ + \text{F}^- \\
\text{HF} + \text{NaOH} & \rightleftharpoons \text{H}_2\text{O} + \text{NaF}
\end{align*}
\]

K = \(1 \times 10^{-6}\)
K = \(6.8 \times 10^{-4}\)
K = \(6.8 \times 10^{10}\)

1. Key: Strong acid H⁺ can be consumed by weak base.
   • Strong acid is replaced by weak acid
     HCl in, NH₄⁺ out
   2. Strong base HO⁻ can be consumed by weak acid
     • The strong base is replaced by weak base
       NaOH in, NaF out
B. **Buffer Action**: A buffer system contains both a weak acid (to neutralize any added OH\(^{-}\)) and a weak base (to neutralize any added H\(^{+}\)) (Fig 17.2 Buffer Action)

- normally the weak acid and weak base are conjugates
- normally they are present in approximately equal quantities, but not always

1. Since a strong acid reacts with a weak base to produce a weak acid, addition of strong acid results in a weak increase in acidity rather than a strong increase
2. Since a strong base reacts with a weak acid to produce a weak base, addition of strong base results in a weak increase in basicity rather than a strong increase

C. The pH of buffer solutions: Ka, pKa, H\(^{+}\), pH

1. **Formula**

\[
HA \rightarrow H^{+} + A^{-} \quad K_a = \frac{[H^+][A^-]}{[HA]}
\]

rearrange: \[pH = pK_a + \log \frac{\text{base}}{\text{acid}}\] “Henderson-Hasselbalch Equation”

\[pK_a = -\log K_a\]
\[K_a = 10^{-pK_a}\]

**Practical note**: the base/acid ratio can be entered in moles/moles or in molarity/molarity, whichever is easier and more accessible
• since the volume is common to both base and acid

2. **Some Practical Simple Stuff**
   a. A buffer pH depends on:
      1. pK\(_a\) which depends on K\(_a\) (weak acid strength)
      2. Base/acid ratio
   b. If you have a target pH that you want for a solution, choose an acid with the appropriate K\(_a\) and pK\(_a\)
   c. If the base/acid ratio = 1, then pH = pK\(_a\)
      • This is a helpful reference: basically the pH ≈ pK\(_a\), adjusted to the basic or acidic side of pK\(_a\) depending on whether you have more of the base or of the acid
        a. More base than acid: \(\rightarrow\) pH number is higher (more basic) than pK\(_a\)
        b. Less base than acid: \(\rightarrow\) pH number is lower (more acidic) than pK\(_a\)
   d. Adding base increases the weak base/weak acid ratio, and makes the pH more basic
   e. Adding acid decreases the weak base/weak acid ratio, and makes the pH more acidic

C. **Buffer Capacity**: determined by the quantity of weak acid/base
   -dictates how much strong acid/base is consumed before buffer exhausted
   -if more strong acid(base) is added than there is weak base(acid), the capacity is exceeded.
Consider 1.0 L water. Determine the pH’s for the following solutions.

The $K_a$ for HF = $6.8 \times 10^{-4}$

1. A solution with 0.5 moles of HF?

2. A solution with 0.5 moles of NaF?

3. A solution with 0.5 moles of HF and 0.5 moles of NaF?

4. A solution with 0.5 moles of HF and 0.5 moles of NaF, to which 0.1 moles of HCl is added?

5. A solution with 0.5 moles of HF and 0.5 moles of NaF, to which 0.1 moles of NaOH is added?

Tips:
1. Diagnose the scenario
2. For a buffer, use Henderson-Hasselbalch equation
3. When strong acid or base is added to a buffer, write out the reaction that would occur with the strong acid or base on the left side, and then work out the ICE change.
D. Creating/Recognizing a Buffer
   - must end with **weak acid/weak base** pair

1. Mix some weak acid with it’s conjugate weak base

   HF, NaF   H₃PO₄, NaH₂PO₄   NaHSO₄ ,Na₂SO₄   CH₃COOH, CH₃COONa   NH₄Cl, NH₃

2. Start with one of conjugate pair, and add less than an equivalent amount of strong acid or strong base to create the other

<table>
<thead>
<tr>
<th>Begin</th>
<th>Add (1/2 as much)</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weak Acid</td>
<td>Strong Base</td>
<td>weak acid + conj base</td>
</tr>
<tr>
<td>Weak Base</td>
<td>Strong Acid</td>
<td>weak acid + conj base</td>
</tr>
</tbody>
</table>

   Ex:
   HF + NaOH → H₂O + NaF
   I 0.4 moles 0.20 moles 0

   C

   E

   Ex:
   NH₃ + HCl → NH₄⁺Cl⁻
   I 0.6 mol 0.3 mol 0

   C

   E

   Note: doesn’t work if you add too much strong acid/base:

   Ex:
   NH₃ + HCl → NH₄⁺Cl⁻
   I 0.6 mol 0.6 mol 0

   C

   E
Buffer Recognition
1. Which combos would make a buffer solution?

a. HF, KF  
e. HN₃, NaCl⁻  
b. NH₄Cl, NH₃  
f. KF, NaF  
c. HCl (0.2 M), NaN₃ (0.4 M)  
g. KF, NaOH  
d. HCl (0.2 mol), NaN₃ (0.1 M)  
h. HF, HCl  
i. NH₃, KF  
j. HF (0.2 mol), NaOH (0.1 mol)

Key:  
a) two acids don’t work  
b) two bases don’t work  
c) an acid and a base is required  
d) tricky: strong and weak mixtures react, sometimes resulting in a buffer combination (both weak acid and weak base, examples c and j), but sometimes not, depending on ratios (example d)

2. Which could be added to 0.4 mol Na⁺CN⁻ to give a buffer?

a. HCN  
b. 0.2 mol HCl  
c. 0.2 mol NaOH  
d. 0.5 mol HCl

3. Which could be added to 0.4 mol H₃PO₄ to give a buffer?

a. NaH₂PO₄  
b. 0.2 mol NaOH  
c. 0.4 mol NaOH  
d. 0.2 mol HCl  
e. KCl
E. Simple Buffer Calc
   a. find pH  Logic:  \( K_a \rightarrow pK_a \rightarrow pH \)
   b. find \( K_a \)  Logic:  \( pH \rightarrow pK_a \rightarrow K_a \)
   c. solve for quantity of acid or base needed for target pH
      Logic:  \( pH \) and \( pK_a \)  \( \rightarrow \)  base/acid ratio  \( \rightarrow \)  needed quantity to achieve that ratio

\[
pH = pK_a + \log \frac{[\text{base}]}{[\text{acid}]} \]

“Henderson-Hasselbalch Equation”

•  remember:  mole/mole ration works as well as molarity/molarity ratio for base/acid

1. **Finding pH**  Find pH for buffer 0.12 M HNO\(_2\), 0.16M NaNO\(_2\)  \( K_a(\text{HNO}_2) = 4.5 \times 10^{-4} \)

2. **Finding pK\(_a\)**  \([\text{HA}] = 0.11 \text{ M} \quad [\text{NaA}] = 0.13 \text{ M} \). What is \( K_a \) for HA if pH = 5.18?

3. **Finding quantity of acid or base**  How many grams of NaF (42 g/mol) should be added to 612 mL solution of 0.4 M HF (\( K_a = 6.8 \times 10^{-4} \)) to produce a buffer solution in which pH = 3.10?

   Logic:
F. pH Changes after Strong Acid or Strong Base is added to a Buffer Solution
(note: the following is written in terms of adding strong acid. Use analogous strategy for addition of strong base)

1. Write out the reaction of the added strong acid with the buffer’s weak base
2. Make out an ICE-type table to show how the quantities of weak base and weak acid change as a result of the reaction
   a. Remember that a strong acid basically reacts completely with the weak base
   b. It’s often best to do the ICE in terms of moles rather than molarity
   c. The “E” at the end is really the “End” of the acid-base reaction. It may not represent equilibrium
3. Assess the situation at the end of the ICE analysis
   a. Are both weak acid and weak base present?
      • buffer situation ⇒ Use HH equation
        \[ \text{pH} = \text{pK}_a + \log \frac{[\text{base}]}{[\text{acid}]} \]
   b. Is only weak acid left? (the strong acid exhausted the weak base?)
      • weak acid situation ⇒ use Ch. 16 procedure.
      • Be sure your weak acid is in terms of molarity, and factors in the combined volumes
        \[ [\text{H}^+] = \sqrt{\text{K}_a \times [\text{weak acid}]} \]
   c. Is strong acid as well as weak acid left? (The strong acid exhausted the buffer’s weak base, and you still have some strong acid left over?)
      • Strong acid situation
      • Be sure your strong acid is in terms of molarity, and factors in the combined volumes
        \[ [\text{H}^+] = [\text{strong acid}] \]

Note: Scenario “a” is when the buffer capacity is not exhausted, and you still have a buffer situation at the end. Scenario’s “b” and “c” both represent cases where the buffer capacity is exhausted, and you do not have a buffer scenario at the end of the reaction.

Problems
Given a 1.0 L solution that is initially 0.2 M in both HF and KF, assess the situation and find the pH after the following acids or bases are added.

a. 0.1 mol HCl added

Assess:

Reaction:

I

C

E
b. 0.1 mol NaOH added  Assess:

Reaction:

I

C

E

c. 1.0 L of 0.20-M HCl added  Assess:

Reaction:

I

C

E

d. 1.5 L of 0.20-M mol NaOH added  Assess:

Reaction:

I

C

E

17.2 Acid-Base Titration

1. An acid and a base are mixed, at least one being strong
2. “equivalence point” or “end point:” equal moles of acid/base
3. Indicator often used to turn color, or pH meter used to give “titration curve”
4. Unlike buffer, pH changes fast near endpoints

A. Endpoint Situations: Qualitative, Non-Math Version

<table>
<thead>
<tr>
<th>Acid In</th>
<th>Base In</th>
<th>Conjugate Acid Out</th>
<th>Conjugate Base Out</th>
<th>Endpoint Outcome</th>
<th>Endpoint pH</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Acid</td>
<td>Strong Base</td>
<td>Non-acid</td>
<td>Non-base</td>
<td>Neutral</td>
<td>pH = 7.0</td>
<td>1</td>
</tr>
<tr>
<td>Strong Acid</td>
<td>Weak Base</td>
<td>Weak acid</td>
<td>Non-base</td>
<td>Weakly acidic</td>
<td>pH &lt; 7.0</td>
<td>2</td>
</tr>
<tr>
<td>Weak Acid</td>
<td>Strong Base</td>
<td>Non-acid</td>
<td>Weak base</td>
<td>Weakly basic</td>
<td>pH &gt; 7.0</td>
<td>3</td>
</tr>
</tbody>
</table>

1. Would the pH at endpoint be acidic, basic, or neutral when titrated with NaOH?
   a. HCN
   b. HNO₃
   c. HF

2. Would pH at endpoint be acidic, basic, or neutral when titrated with HCl?
   a. NaOH
   b. NaF⁻
B. Calculation of pH at the **Endpoint** for a Titration

- Review: at endpoint equal moles of acid and base have been added. At least one of them was a strong acid or base, and possibly both.

- Three Possible Scenarios

<table>
<thead>
<tr>
<th>Acid In</th>
<th>Base In</th>
<th>Endpoint Outcome</th>
<th>Endpoint pH</th>
<th>pH Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Acid</td>
<td>Strong Base</td>
<td>Neutral</td>
<td>pH = 7.0</td>
<td>None needed, pH = 7</td>
</tr>
<tr>
<td>Strong Acid</td>
<td>Weak Base</td>
<td>Weak acid</td>
<td>pH &lt; 7.0</td>
<td>( K_a \rightarrow [H^+] \rightarrow \text{pH} )</td>
</tr>
<tr>
<td>Weak Acid</td>
<td>Strong Base</td>
<td>Weak base</td>
<td>pH &gt; 7.0</td>
<td>( K_a \rightarrow K_b \rightarrow [OH^-] \rightarrow \text{pOH} \rightarrow \text{pH} )</td>
</tr>
</tbody>
</table>

**Problem:** What is the pH after 40 mL of 0.10-M NaOH is added to 20 mL of 0.20-M HClO? (The \( K_a \) for HClO = 3.0 x 10\(^{-8}\))

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**C. Titration Curves:** a plot of how pH changes as strong acid is added to a solution of base, or as a strong base is added to a solution of acid
2 Key Diagnostic features:
1. Initial pH
   - Acidic or basic? Strong or weak?
   - This reflects what you started with, before you titrate it with the strong acid or base.
2. The endpoint/equivalence point pH
   - Acidic, neutral, or basic?
   - As we saw before, this reflects what kind of acid and base were involved

<table>
<thead>
<tr>
<th>Initial pH</th>
<th>pH &lt; 2</th>
<th>2 &lt; pH &lt; 7</th>
<th>7 &lt; pH &lt; 12</th>
<th>12 &lt; pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial acid/base</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Endpoint pH</th>
<th>pH &lt; 2</th>
<th>pH = 7</th>
<th>pH &gt; 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Odds n ends
1. Endpoint pH is where pH changes most steeply, rapidly
2. When a weak acid is titrated by a strong base, the titration involves an extended “buffer zone” prior to the end-point, during which the pH change is rather gradual
   a. Prior to the endpoint, some of the weak acid is converted to weak base, so a conjugate weak acid/weak base buffer situation exists.
   b. The analogous situation happens when a weak base is titrated by a strong acid
3. When a weak acid “HA” is titrated by a strong base, at the half-way point (half-way to the end-point), half of the original weak acid HA is converted to its weak conjugate base A⁻
   - At this point, the pH = pK_a  (see HH-equation when [base]/[acid] = 1)
   - The analogous situation happens when a weak base is titrated by a strong acid

Problems
1. An initial pH = 1.3 and an equivalence point of 7 corresponds to a titration curve in which a _______ is added to a ____________________.

2. An initial pH = 9.3 and an equivalence point of 4.7 corresponds to a titration curve in which a _______ is added to a ____________________.
D. Titration Calculations Involving Endpoint/Equivalence Point

Key: At equivalence point, \( \text{moles acid} = \text{moles base} \)
- know one \( \Rightarrow \) know other

Applications
1. Find volume required
2. Determine unknown concentrations
3. Determine unknown grams

Problems
1. What volume of 0.12 M NaOH is needed to titrate 36 mL of 0.14 M HCl to its endpoint?

2. When 42 mL of aqueous HCl is titrated by 0.10 M NaOH, it takes 25 mL of the NaOH solution to reach the endpoint. What is the [HCl] of the original solution?

3. How many grams of KOH (56 g/mol) would it take to neutralize 86 mL of 1.2 M HNO₃?
E. General pH Strategy: Finding the pH after Acid or Base is added to a solution

- Whether it be to a buffer solution, a strong acid solution, a weak acid solution, a strong base solution or a weak base solution prior to the new addition
- The strategy is similar to when we added acid or base to a buffer

1. Write out the acid-base reaction
   - It helps to recognize which species are acidic or basic or neutral, and weak or strong

2. Calculate the initial moles
   - Number of moles = molarity x volume (in Liters)

3. Use ICE to determine post-reaction (“E”) moles

4. Assess the post-reaction situation, based on what's left at the end

5. Solve the pH problem from there

| Post-Reaction Situation                              | Formula/Equation                                                                 |
|------------------------------------------------------|********************************************************************************|
| 1 Strong acid only                                   | [H^+] = [SA]                                                                     |
| 2 Strong acid plus weak acid                         | [H^+] = [SA]                                                                     |
|                                                       | Ignore WA, which makes insignificant contribution                                |
| 3 Strong base only                                   | [HO^-] = [SB]                                                                    |
| 4 Strong base plus weak base                         | [HO^-] = [SB]                                                                    |
|                                                       | Ignore WB, which makes insignificant contribution                                |
| 5 Weak acid only                                     | [H^+] = \sqrt{K_b x [WB]}                                                      |
|                                                       | Qual: pH < 7                                                                    |
| 6 Weak base only                                     | [HO^-] = \sqrt{K_b x [WB]}                                                      |
|                                                       | Qual: pH > 7                                                                    |
|                                                       | May need to find K_b from K_a                                                  |
| 7 Weak acid plus weak base                           | pH = pK_a + log \left( \frac{[\text{base}]}{[\text{acid}]} \right)            |
|                                                       | Buffer solution                                                                |
| 8 No acid or base; only neutral salts               | pH = 7.0                                                                        |

- For many of these, it will be necessary to calculate molarities
- To do this, make sure that you factor in the total, combined volume

**Key:** Recognizing the Final Situation!!
Strong base added to weak acid. (Strong acid/weak base combinations would be analogous.)

What is pH after:

A. 20 mL of 0.10 M NaOH is added to 20 mL of 0.10 M HClO? (Ka = 3.0 \times 10^{-8})

B. 20 mL of 0.10 M NaOH is added to 10 mL of 0.10 M HClO?

C. 20 mL of 0.10 M NaOH is added to 30 mL of 0.10 M HClO? (Ka = 3.0 \times 10^{-8})
F. Addition of strong acid to a weak acid solution. (Analog would be strong base to weak base)

1. A solution has 1 L of 0.40 M acetic acid ($K_a = 1.80 \times 10^{-5}$). Find the pH and $[\text{CH}_3\text{COO}^-]$

Ex: $\text{HA} + \rightleftharpoons \text{H}^+ + \text{A}^-$

2. Suppose 0.10 moles of HCl is added to the above solution (assume no change in volume). Find the pH and $[\text{CH}_3\text{COO}^-]$

Ex: $\text{HA} + \rightleftharpoons \text{H}^+ + \text{A}^-$

Notice:
1. The strong acid dominates the pH
2. When a weak acid is in water by itself, then $[\text{H}^+] = [\text{A}^-]$
3. But when strong acid was added (as an additional source of $[\text{H}^+]$), $[\text{H}^+] >> [\text{A}^-]$
4. The equilibrium constant still applies. With $[\text{H}^+]$ up, $[\text{A}^-]$ goes down, ala LeChatelier’s principle