This miniproject asks you to find and explain the error in reasoning in three proofs. For two of them you will need the following information.

A Pythagorean triple is an ordered triple of integers (a, b, c) such that $a^2 + b^2 = c^2$. Note that if (a, b, c) is a Pythagorean triple, so is (b, a, c). Similarly, a Pythagorean quadruple is an ordered four-tuple (a, b, c, d) such that $a^2 + b^2 + c^2 = d^2$. As an example, note that

(3, 4, 5) is a Pythagorean triple since $3^2 + 4^2 = 5^2$,

(5, 12, 13) is a Pythagorean triple since $5^2 + 12^2 = 13^2$,

and (3, 4, 12, 13) is a Pythagorean quadruple since $13^2 = 5^2 + 12^2 = (3^2 + 4^2) + 12^2 = 3^2 + 4^2 + 12^2$.

- (a) Explain why the argument given in #34 of Section 1.6 of the textbook does not work.
- (b) Consider the following theorem and the two proofs given.

Theorem For every Pythagorean quadruple (a, b, c, d) there exists some order of the arguments a, b, c and an integer e such that (a, b, e) and (e, c, d) are both Pythagorean triples.

Proof 1 Since (a, b, c, d) is a Pythagorean quadruple, we know that

$$a^2 + b^2 + c^2 = d^2.$$

But, by subtracting c^2 for both sides, this tells us that

$$a^2 + b^2 = d^2 - c^2$$

Therefore, let $e^2 = d^2 - c^2$. Then we have both of the following

$$a^{2} + b^{2} = e^{2}$$
 (by definition of e), and
 $e^{2} + c^{2} = (d^{2} - c^{2}) + c^{2} = d^{2}$,

which proves the theorem.

Proof 2 Since (a, b, c, d) is a Pythagorean quadruple, we know that

$$a^2 + b^2 + c^2 = d^2.$$

Assume that c is the largest of the three integers, so $c \ge a$ and $c \ge b$, so that $a^2 + b^2 = c^2$. Then for e = c we have

$$a^{2} + b^{2} = c^{2} = e^{2}$$

and $e^{2} + c^{2} = a^{2} + b^{2} + c^{2} = d^{2}$

which is what we wished to prove.

Note that *neither* proof is correct. Do each of the following.

- (i) Find and explain the flaw of reasoning in the first proof.
- (ii) Find and explain the flaw of reasoning in the second proof.
- (iii) Is the theorem really a theorem? That is, is it correct? Explain.

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