

This project asks you to think about Hamiltonian circuits in the context of how a knight moves on a chessboard.

A knight in the game of chess can move in L-shaped paths: either two squares vertically and one square horizontally, or one square vertically and two squares horizontally. So a knight in the center of the chessboard has eight possible moves: 2 forward and 1 right, 2 forward and 1 left, 2 backward and 1 right, 2 backward and 1 left, 1 forward and 2 left, etc..

A “Knight’s tour” is a sequence of moves that the knight takes on a chessboard in such a way that it reaches every square on the chessboard. In other words, if you view the squares on the chessboard as the vertices of the graph, a Knight’s Tour is a Hamiltonian path. A “Reentrant Knight’s Tour” is a knight’s tour that is a Hamiltonian circuit, rather than just a path, so that it returns to the square from which it started.

- (a) Find a Knight’s Tour on a  $3 \times 4$  chessboard (3 rows and 4 columns).
- (b) Show that there is no Reentrant Knight’s Tour on a  $3 \times 4$  chessboard. (Hint: Start with those squares on the board for which a knight would have only two moves. The edges corresponding to those moves must be included in the circuit. Build from there and see what happens.)
- (c) Show that there is no Knight’s Tour on a  $3 \times 3$  chessboard.
- (d) Is there a Reentrant Knight’s Tour on a  $4 \times 4$  chessboard? If so, show it. If not, explain why not.