This miniproject asks you to justify some standard rules of inference that will later help you in proving more complicated and specific propositions.

Give formal two-column proofs of each of the following.

(a) Universal Modus Tollens

$$\frac{\forall x \left(P(x) \to Q(x) \right)}{\neg Q(a)}$$
$$\frac{\neg P(a)}{\Box \neg P(a)}$$

(b) Universal Transitivity

$$\begin{array}{c} \forall x \left(P(x) \to Q(x) \right) \\ \forall x \left(Q(x) \to R(x) \right) \\ \hline \therefore \forall x \left(P(x) \to R(x) \right) \end{array}$$

$$\frac{\forall x \left(P(x) \to \left(Q(x) \land S(x) \right) \right)}{\forall x \left(P(x) \land R(x) \right)}$$
$$\therefore \forall x \left(R(x) \land S(x) \right)$$

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