

This miniproject asks you to justify some standard rules of inference that will later help you in proving more complicated and specific propositions.

Give formal two-column proofs of each of the following.

(a) Universal Modus Tollens

$$\frac{\begin{array}{l} \forall x (P(x) \rightarrow Q(x)) \\ \neg Q(a) \end{array}}{\therefore \neg P(a)}$$

(b) Universal Transitivity

$$\frac{\begin{array}{l} \forall x (P(x) \rightarrow Q(x)) \\ \forall x (Q(x) \rightarrow R(x)) \end{array}}{\therefore \forall x (P(x) \rightarrow R(x))}$$

(c)

$$\frac{\begin{array}{l} \forall x (P(x) \rightarrow (Q(x) \wedge S(x))) \\ \forall x (P(x) \wedge R(x)) \end{array}}{\therefore \forall x (R(x) \wedge S(x))}$$