This sheet does not have room to work on it, so please submit your completed assignment on your own paper.

1. (a) Assume $x = e^t$ so that $t = \ln(x)$ on the interval $I = (0, \infty)$.
   i. (1.5 points) Show that $\dfrac{dy}{dx} = \dfrac{1}{x}\dfrac{dy}{dt}$.
   ii. (3 points) Show that $\dfrac{d^2y}{dx^2} = \dfrac{1}{x^2}\dfrac{d^2y}{dt^2} - \dfrac{1}{x^2}\dfrac{dy}{dt}$.
   Hint: $\dfrac{d^2y}{dx^2} = \dfrac{d\left(\frac{dy}{dx}\right)}{dx} = \dfrac{d\left(\frac{dy}{dx}\right)}{dt} \cdot \dfrac{dt}{dx}$ Then use part (i) and the product rule.
   iii. (1 point) Rewrite the differential equation
   $x^2\dfrac{d^2y}{dx^2} - 6y = 0$
   in terms of y and t instead of y and x.
   iv. (3 points) Solve the differential equation for $y(t)$.
   v. (1 point) Substitute $t = \ln(x)$ to find $y(x)$. Simplify appropriately.

   (b) (2.5 points) Solve the differential equation
   $x^2\dfrac{d^2y}{dx^2} - 6y = 0$
   using the Cauchy-Euler approach (of assuming the answer is of the form $y = x^m$) instead.

2. (4 points each) Solve the homogeneous Cauchy-Euler differential equations below.
   (a) $x^3y^{(3)} + x^2y'' - 2y = 0$
   (b) $x^3y^{(3)} - 12xy' + 24y = 0$
   (c) $x^4y^{(4)} + 2x^3y^{(3)} + 5x^2y'' + 19xy' - 39y = 0$
   Hint: For (c), one of the solutions to the auxiliary equation is $m = \sqrt{3}$.

3. (8 points) Solve the following non-homogeneous Cauchy-Euler differential equation.
   $2x^2y'' + 5xy' + y = x^2 - x$

4. Consider the differential equation
   $(x-5)^2\dfrac{d^2y}{dx^2} - 8(x-5)\dfrac{dy}{dx} + 14y = 0.$
   (a) Solution Method A:
   i. (1.5 points) Use the substitution $u = x - 5$ to rewrite the differential equation in terms of y and u instead of y and x.
   ii. (2.5 points) Solve the differential equation from part (i) for $y(u)$.
   iii. (1 point) Find $y(x)$.

   (b) (3 points) Solution Method B: Considering the original differential equation in y and x, use the guess of $y = (x-5)^m$ to solve the differential equation.