

This miniproject asks you about the “ $3x + 1$ Conjecture” described in the book (see Example 23).

(a) Verify the “ $3x + 1$ Conjecture” for the following integers.

- (i) 6
- (ii) 7
- (iii) 15
- (iv) 17
- (v) 21
- (vi) 30
- (vii) 153
- (viii) 1024

(b) Prove the following:

Let M be a positive integer. Suppose that for every positive integer k with $k \leq M$, we know that there is some number of repetitions of the transformation T , applied to k , that will result in the integer 1. Then there is some number of repetitions of the transformation T , applied to $2M$, that will reach the integer 1.

(c) Prove the following:

Let M be a positive integer. Suppose that for every positive integer k with $k \leq M$, we know that there is some number of repetitions of the transformation T , applied to k , that will result in the integer 1. Then there is some number of repetitions of the transformation T , applied to $4M$, that will reach the integer 1.

(d) Prove the following:

Let M be a positive integer. Suppose that for every positive integer k with $k \leq M$, we know that there is some number of repetitions of the transformation T , applied to k , that will result in the integer 1. Then there is some number of repetitions of the transformation T , applied to $2^n M$ for every positive integer n , that will reach the integer 1.