There is a pencil-puzzle known by several different names, some of which that I've come across are "Paint-doku", "Pencil Pixel Puzzles", "Picture Puzzles", "O'Ekaki", and "Nonograms". Since the first term that I came across for this kind of puzzle is O'Ekaki, that is the term that I am going to use. The puzzle, and some basic strategy, are described in the following pages. In the following, I am going to make a distinction between an O'Ekaki puzzle and an O'Ekaki solution. A puzzle is one where the grid is still blank. The dimensions have been determined, as have the numbers that are the clues at the top of each column and start of each row. A solution is a filled in grid that is consistent with the clues. (Note that in O'Ekaki the puzzles published are in general recognizable pictures, hence some of the names, the puzzles do not have to be restricted to those that make pictures. A lot of them look more like ink-blots either way.)

(a) Consider a 2×2 O'Ekaki grid. Note that there are 16 possible solutions. Prove or disprove the following conjecture: There are 16 puzzles, each of which has a unique solution.

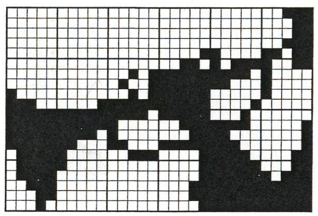
(b) Form a conjecture for the number of possible O'Ekaki solutions for a general grid of size $m \times n$. Prove your conjecture.

(c) Find a counterexample to the following conjecture: Every 5×5 O'Ekaki puzzle has a unique solution.

"The Mind-Challenge Puzzle Book"
Ed. by Emily Cox, Henry Rathvon, Henry Hook, Paul Sloane, and Des MacHale
2002

Perplexing Pixel Puzzles

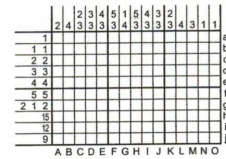
Visual Challenges for the Logical Mind



Introduction

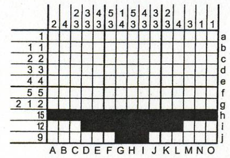
Whether or not you have any artistic talent, you can draw the pictures in this book. All that is required is a pencil and a logical mind.

In a Pixel Puzzle, the numbers along the top and left side of the diagram are all you need to figure out which squares should be colored in and which should be left blank. The simplest way to understand how this works is to look at an example. Check out the puzzle below. Along the bottom and the right side are letters that we'll use to refer to the columns and rows. So B is the second column from the left, and c is the third row from the top. Square jO is the lower right box.

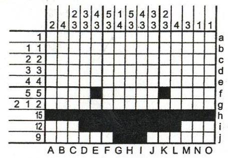


Look at column L, the twelfth column. (The lines separating every fifth row and column are slightly thicker to make for easier counting.) At the top of the column is a "4." This means that when the puzzle is correctly solved, there will be a group of four consecutive squares in column L that are black, and all the remaining squares will be empty. Similarly, in row i, there will be twelve consecutive black squares, and the remaining three squares will be empty. Now look at row e. It says "4 4." This means that there are two separate groups of four black squares in the column, separated by at least one white square. In column F, there are two separate groups of black squares, one with five black squares, and one with three. The groups of black squares always follow the same sequence as the numbers. So in column F, the group of five black squares will be above the group of three black squares. In row g, there will be, from left to right, a group of two black squares, a single black square, and another group of two black squares, and these three groups will be separated from each other by at least one white square. The trick to solving is to determine which squares you know for certain are black and which are definitely white.

Check out row h. It has a total of 15 squares in it, and the number on the left-says "15." It doesn't get much easier than this. All 15 squares must be black. (If a row is entirely empty, it's marked with a zero.) Now look at row i. Somewhere in the row, there are 12 consecutive black squares. If they start all the way on the left, they'll go from square iA through square iL. There are three other possibilities: It could go from iB to iM, iC to iN, or iD to iO. But in all four of the possible locations for the group of 12 black squares, the squares from iD through iL are filled in. So we can confidently blacken those nine squares. Row j is similar. If the group of nine black squares is as far left as possible, it will extend right to column I, while if it went as far right as possible, it would extend left to column G. In any case, the central three squares in row j (jG, jH, and jI) must be black. So we can fill those in. Whenever there is a single number in a row or column that is greater than half the number of squares in that row or column, you can always determine that one or more squares must be black. Our puzzle now looks like this:



Now look at row f. It has two groups of black squares, both five squares in length. These two groups must, of course, be separated by at least one white space. Where can the left group of black squares go? Well, it can go in squares fA to fE, fB to fF, fC to fG, fD to fH, or fE to fI. Can it go in squares fF to fJ? No! The reason it can't is because that would leave only five white squares to the right of it, not enough to put the group of five black squares with the mandatory white space in between. So looking at the five places where the left group of five black squares can go, we see that square fE is always black, so we can fill it in. Likewise, no matter where the right group of five goes, square fK must be a part of it, so we can fill that in. Now the puzzle looks like this:



Let's look at some columns now. Column K already has three of its squares blackened. When completed, it will have a group of two black squares above a group of three black squares. What would happen if square gK were filled in? It would create a group of four consecutive black squares, which can't be the case. So square gK must be empty. We indicate this by drawing a little "×" in it. (Some solvers prefer to use a dot.) The × in square gK gives us lots of information. We now know the entire appearance of the row. The group of three black squares must be hK, iK, and jK, and the group of two black squares must be fK and eK. The remaining squares all have to be empty. Column E is similar. If square gE were black, a group of four black squares would be formed. But since there would be only one white square below that group of four black squares, there wouldn't be enough room to form a group of three black squares, as indicated by the numbers at the top of the column. So square gE is white. Once it's marked with an ×, the remaining squares in the column are simple. Square jE must be black to form the

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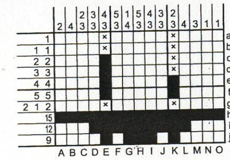
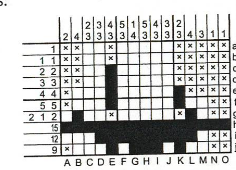
group of three black squares, and squares eE, dE, and cE must be blackened to make the group of four black squares. The remaining two squares are empty. The puzzle now looks like this:



Look at columns N and O. Both say "1" at the top, and both have one square already blackened. Therefore, all the remaining squares must be white. In column M, there is one black square already, so that square must be part of the group of three black squares. Thus, since hM is part of the group of three black squares, the group can't possibly extend up beyond square fM, so all the squares above that are empty. And in column L, square gL must be black, because if it weren't, there wouldn't be enough space to have four consecutive black squares. Once you blacken square gL, you know that either fL or jL contains the final black square in that column. You don't know yet which, but you do know that the top fives squares of the column must all be white. Similar logic in columns A and B result in the puzzle looking like this:



Now look at row j. There are three separate groups of black squares. But since there's only one number at the left of the column (a "9"), we know that any white squares that come between black squares must be blackened. So fill in squares jF and jJ. In column F, now, there are three consecutive black squares at the bottom of the column. This corresponds to the "3" in column F, so the square above the group (gF) is white. Likewise, squares gH, gI, and gJ are all white. In column G, the bottom number at the top of the column is a "4." This means that the group of three black squares must extend one square farther, and then have a white square above that. Following all of this yields a puzzle that looks like this:

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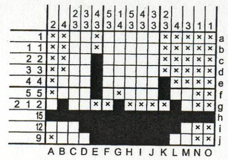
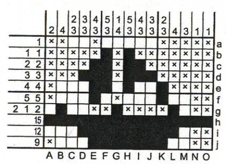
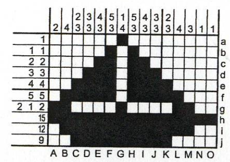


We're almost done! Go back to column F. There are six white squares that need to hold a group of five black squares. No matter how they go in, the four middle squares must be black. So blacken squares bF, cF, dF, and eF. In the same vein, in column H, squares bH, cH, dH, and eH must be black, and in column I, no matter how the four black squares go in the six white spaces, squares cI and dI will always be black. The black squares we've just added complete rows b and c, so all the remaining squares can be marked with an ×. In row d, square dG can't be black, since that would make a group of five black squares, so dG is white, which makes dJ black, dD black, and dC white. Here's what we've reached:



By now you should be able to finish, but in case you're having trouble, follow along. First, eJ and eI must be black. So eG must be white. Therefore, eD and eC are black, and eB, white. Now looking at column G, aG is black, so the rest of row a is white. This forces fF, fH, fI, and fJ to be black. Examining row f, we see that fL is black, fM is white, and fD, fC, and fB are black. In column B, iB and jB need to be black, while in columns C and D, gC and gD are white, while iC, jC, and jD are all black. This lets us finish row g with black squares in gA and gM, which in turn tell us that column A is finished, so iA is white, and column M must have a black square in iM and a white square in jM. The final square, jL, is white. Voilà. A sailboat.



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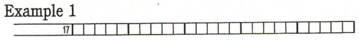
Don't expect the puzzles in this book to be as smooth sailing as this one was. The bigger pictures are much harder. If you encounter points in solving where you think there's no way to logically conclude anything about any unknown squares, just think harder. There's always something to go on. And often, getting just one additional bit of data cascades into much more information. Of course, filling in even one square wrong can lead to disastrous results minutes later when you reach a contradiction and don't know where you went wrong. So be careful not to fill in a square unless you are certain that it's either definitely black or definitely white.

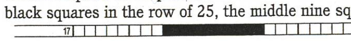
If you finish this book and need more Pixel Puzzles to solve, try the companion book in this series, *Mind-Sharpening Pixel Puzzles*.

Every decision when solving a Pixel Puzzle is based on analyzing a row or a column, one at a time, and making logical decisions as to which squares should be blackened and which should be marked with an ×. These decisions depend on the numbers and the present status of the specific row or column being analyzed. Below are a few sample rows that show the logic you might encounter while solving the puzzles.

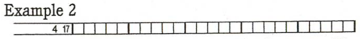
Example 1



The block of 17 is wider than half of the width of the 25-square row. This creates the nine-square overlap shown below. No matter how you place the 17 consecutive black squares in the row of 25, the middle nine squares must be black.

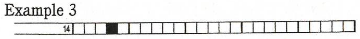


Example 2

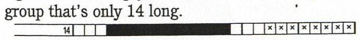


If the group of four squares is as far left as possible, the group of 17 squares would start in the sixth square (the fifth square would be white). That means that it would extend to the 22nd square. If the group of 17 squares started all the way right, it would extend from the right to the ninth square. So squares 9 through 22 are black. Given that square 9 is part of the group of 17, the group of four must fit in the first seven squares. There's no way to do this without the fourth square's being black.

Example 3



Since square 4 is filled in, the group of 14 must extend from there to the 14th square, at least. So everything in between is filled in, and the eight squares on the right must be empty, because there's no way to connect them to square 4 with a group that's only 14 long.



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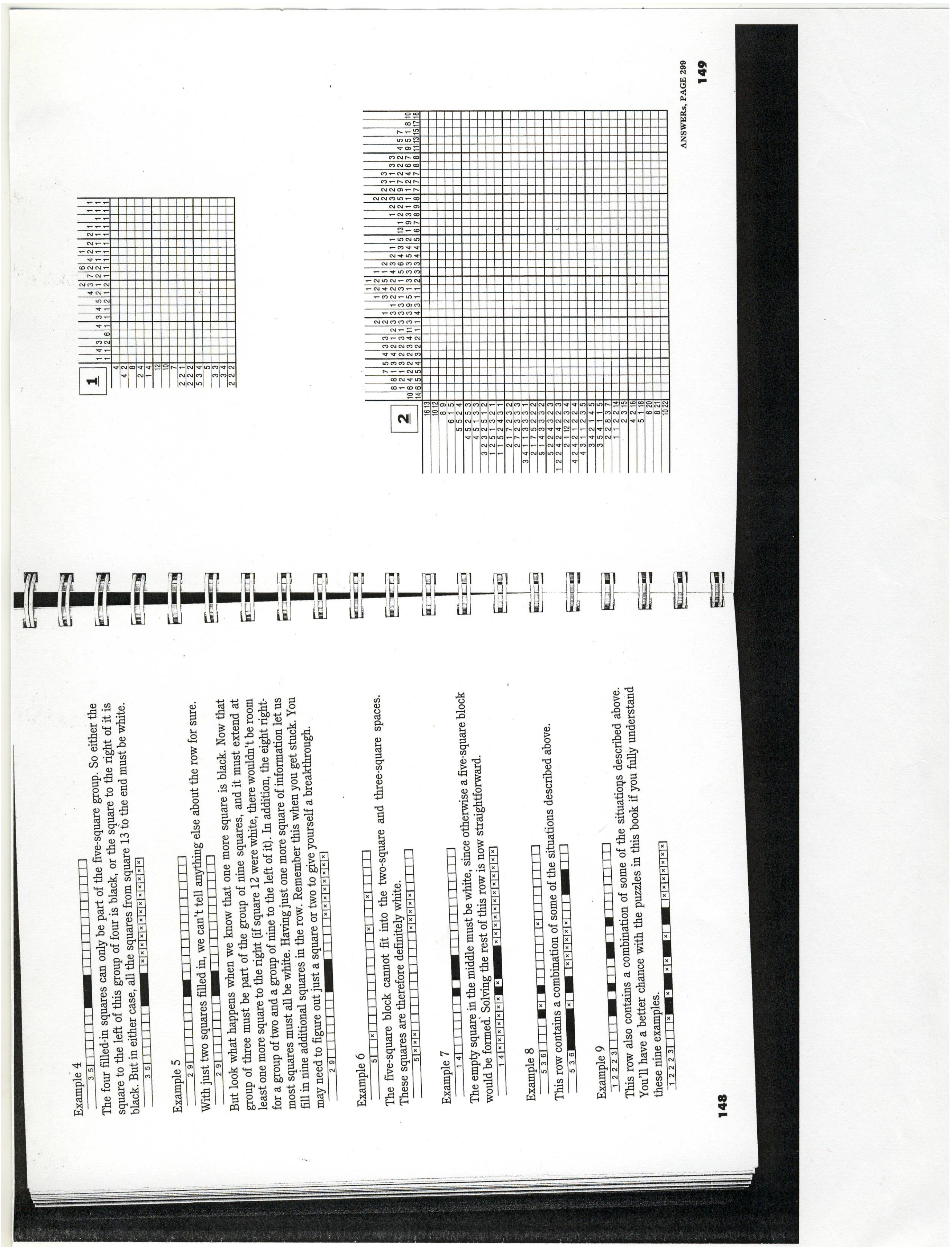

Example 4

The four filled-in squares can only be part of the five-square group. So either the square to the left of this group of four is black, or the square to the right of it is black. But in either case, all the squares from square 13 to the end must be white.

Example 5

With just two squares filled in, we can't tell anything else about the row for sure.

But look what happens when we know that one more square is black. Now that group of three must be part of the group of nine squares, and it must extend at least one more square to the right (if square 12 were white, there wouldn't be room for a group of two and a group of nine to the left of it). In addition, the eight rightmost squares must all be white. Having just one more square of information let us fill in nine additional squares in the row. Remember this when you get stuck. You may need to figure out just a square or two to give yourself a breakthrough.

Example 6

The five-square block cannot fit into the two-square and three-square spaces. These squares are therefore definitely white.

Example 7

The empty square in the middle must be white, since otherwise a five-square block would be formed. Solving the rest of this row is now straightforward.

Example 8

This row contains a combination of some of the situations described above.

Example 9

This row also contains a combination of some of the situations described above. You'll have a better chance with the puzzles in this book if you fully understand these nine examples.

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