This miniproject asks you to solve a puzzle that I found in the book "536 Puzzles & Curious Problems", by Henry Ernest Dudeney (copyright 1967). The puzzle relies heavily on the concept of modular arithmetic, as described in this section.

Solve the following puzzle (and explain your reasoning):

FEEDING THE MONKEYS

A man went to the zoo monkey house with a bag of nuts. He found that if he divided them equally among the eleven monkeys in the first cage he would have one nut over; if he divided them equally among the thirteen monkeys in the second cage there would be eight left; if he divided them among the seventeen monkeys in the last cage three nuts would remain.

He also found that if he divided them equally among the forty-one monkeys in all three cages, or among the monkeys in any two cages, there would always be some left over.

What is the smallest number of nuts that the man could have had in his bag?

If x is any solution to this puzzle, what property, in modular arithmetic notation, must x have?