THE TRAVELING SALESMAN PROBLEM

Theorem

The complete graph with n vertices has $(n-1)!$ Hamilton circuits. Of these, half are repeats of the other half but traveled backward.

Brute Force Algorithm for Solving TSPs

- List all possible Hamilton circuits for the weighted graph.

- For each Hamilton circuit, add up the weights of the edges in the circuit (this total is called the **weight of the circuit**).

- Of all the circuits, the one(s) with the least weight is optimal and therefore a solution to the problem.

Cheapest Link Algorithm

- Pick the edge with the smallest weight first (in case of a tie pick one at random). Mark the edge.

- Pick the next cheapest unmarked edge and mark it unless (a) it closes a smaller circuit or (b) it results in three marked edges coming out of a single vertex. If there are ties, break them arbitrarily.

- Repeat step 2 until the Hamilton circuit is complete.

Nearest Neighbor Algorithm

- Start at home

- Whenever you are in a city, pick the next city to visit from among the ones you haven't visited yet, and of all such cities pick the one that is closest to where you are (this city is called the **nearest neighbor**). In case of a tie, choose at random. Keep doing this until you've visited all the cities.

- From the last city go home.