This miniproject asks you to work with Magic Number Squares. (A historical tidbit I heard in Philadelphia when on a tour said that during political meetings, Benjamin Franklin used to make up Magic Number Squares when he wasn't paying full attention to whoever was speaking at the time. His "doodles" of Magic Number Squares can be found in some of his notes on the meetings. I will admit, however, that I don't know if this is fact or "Urban Legend".)

A Magic Number Square is a table with an equal number of rows and columns where every row, every column, and each of the two long diagonals add up to the same number. For example, a Magic Number Square using the nine numbers from the set $S = \{15, 16, 17, 18, 19, \ldots, 23\}$ is

20	21	16
15	19	23
22	17	18

.

Note that each of the three rows adds up to 57, each of the three columns adds to 57, and each of the two diagonals adds to 57.

(a) Show that the sets of the numbers in each row of any Magic Square (of any size) form a partition of the set S used to create the Magic Square. (For my example, I would be talking about $\{\{20, 21, 16\}, \{15, 19, 23\}, \{22, 17, 18\}\}$.)

(b) Do the sets of the numbers in each column of any Magic Square (of any size) form a partition? Explain why or why not.

(c) Do the sets of the numbers in each diagonal of any Magic Square (of any size) form a partition? Explain why or why not.

(d) Form a magic square of $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. (Hint: You might want to first figure out what the common sum would have to be.)

(e) For your magic square, give a digraph for the equivalence relation that leads to the partition that you formed by your rows.

(f) If the columns form a partition, repeat part (e) for the columns.

(g) If the diagonals form a partition, repeat part (e) for the diagonals.

(h) Does there exist a magic square for *every* set of nine consecutive integers? Explain.

(i) Does there exist a magic square of *every* set of nine integers? Explain.