

This project asks you to think about complete trees and forests.

- (a) A complete  $m$ -ary tree is a full  $m$ -ary tree such that every leaf is at the same level. (In other words, every interior vertex has exactly  $m$  children, and there are no branches that stop “early”.) How many vertices does a complete  $m$ -ary tree of height  $h$  have?
- (b) How many leaves does a complete  $m$ -ary tree of height  $h$  have?
- (c) A forest of trees is a graph with no simple circuits. It’s distinguished from being a tree in that it is not required to be connected. Each of the connected components is called a tree of the forest. How many edges are there in a forest of  $t$  trees containing a total of  $n$  vertices?
- (d) Draw a forest with four trees and 25 vertices.
- (e) How many vertices does a forest with  $t$   $m$ -ary complete trees of height  $h$  have? (In terms of  $t$ ,  $m$ , and  $h$ .)
- (f) How many leaves does a forest with  $t$   $m$ -ary complete trees of height  $h$  have?