This miniproject asks you to justify some standard rules of inference that will later help you in proving more complicated and specific propositions.

Give formal two-column proofs of each of the following.

(a)

$$\begin{array}{r} \forall x\,(P(x) \vee Q(x)) \\ \forall x\,((\neg P(x) \wedge Q(x)) \rightarrow R(x)) \\ \hline \therefore \forall x\,(\neg R(x) \rightarrow P(x)) \end{array}$$

(b)

$$\begin{array}{r} \forall x\,(P(x) \vee Q(x)) \\ \forall x\,(\neg Q(x) \vee S(x)) \\ \forall x\,(R(x) \rightarrow \neg S(x)) \\ \exists x\,(\neg P(x)) \\ \hline \therefore \exists x\,(\neg R(x)) \end{array}$$