This miniproject asks you to create two proofs, where the second proof uses the proposition proved in the first proof. It then asks you to explain (without a formal proof) a variant of the proved proposition.

A game involving two players starts with a single row of coins of various denominations and of some finite length. Players alternate turns, and on each turn a player must take exactly one coin, which must be one of the two end coins. Play continues until there are no more coins to take. The winner is the player who has the highest monetary value at the end.

(a) Assume that you start with $2n$ coins, where n is an integer, and number the coins sequentially from one end of the row to the other. Prove that the first player to play can play in a way as to get all of the even numbered coins. Generalize to the odd numbered coins.

(b) Doug and NaDean play this game, where the initial row has 84 coins. If NaDean goes first, prove that NaDean can play so that she either wins or the game ends in a tie.

(c) Explain why, if you have an odd number of coins, that the second person to play will

  (i) usually have an advantage, but
  (ii) may not always be able to win or tie.

Note: These problems were inspired from a problem in the book "Mathematical Puzzles: A Connoisseur's Collection", by Peter Winkler, copyright 2004.