This miniproject asks you about the "$3x + 1$ Conjecture" described in the book (see Example 23).

(a) Verify the "$3x + 1$ Conjecture" for the following integers.

(i) 6

(ii) 7

(iii) 15

(iv) 17

(v) 21

(vi) 30

(vii) 153

(viii) 1024

(b) Prove the following:
Let M be a positive integer. Suppose that for every positive integer k with $k \leq M$, we know that there is some number of repetitions of the transformation T, applied to k, that will result in the integer 1. Then there is some number of repetitions of the transformation T, applied to $2M$, that will reach the integer 1.

(c) Prove the following:
Let M be a positive integer. Suppose that for every positive integer k with $k \leq M$, we know that there is some number of repetitions of the transformation T, applied to k, that will result in the integer 1. Then there is some number of repetitions of the transformation T, applied to $4M$, that will reach the integer 1.

(d) Prove the following:
Let M be a positive integer. Suppose that for every positive integer k with $k \leq M$, we know that there is some number of repetitions of the transformation T, applied to k, that will result in the integer 1. Then there is some number of repetitions of the transformation T, applied to $2^n M$ for every positive integer n, that will reach the integer 1.