This miniproject asks you to solve a puzzle.

A farmer needs to carry a wolf, a goat, and a cabbage across a river. The farmer only has a small boat, which can carry the farmer and only one object (either an animal or a vegetable). He can cross the river repeatedly. However, if left alone (not in the presence of the farmer), the wolf will eat the goat and the goat will eat the cabbage.

We can describe each state by listing what is on each shore. For example, we can use the ordered pair (FG, WC) for the state where the farmer and the goat are on the first shore and the wolf and the cabbage are on the other shore. The symbol $\emptyset$ is used when nothing is on a shore. So the state $(FWGC, \emptyset)$ is the initial state. The problem is solved if you can get to the state $(\emptyset, FWGC)$.

(a) Find all allowable states of the puzzle. Give your answer as a list of ordered pairs. An allowable state is a state in which nothing is getting eaten (so the wolf and goat are either on opposite sides of the river or are in the presence of the farmer, and similarly for the goat and the cabbage).

(b) Construct a graph such that each vertex of this graph represents an allowable state and the vertices representing two allowable states are connected by an edge if it is possible to move from one state to the other using one trip of the boat.

(c) Explain, in real terms (that is, boats and rivers), why finding a path from the vertex representing $(FWGC, \emptyset)$ to the vertex representing $(\emptyset, FWGC)$ solves the puzzle.

(d) Find two different solutions of the puzzle, each using seven crossings.

(e) Suppose that the farmer must pay a toll of \$1 to cross the river with an animal. Which solution of the puzzle should the farmer use to pay the least total toll?