This project asks you to think about Hamiltonian circuits in the context of how a knight moves on a chessboard.

A knight in the game of chess can move in L-shaped paths: either two squares vertically and one square horizontally, or one square vertically and two squares horizontally. So a knight in the center of the chessboard has eight possible moves: 2 forward and 1 right, 2 forward and 1 left, 2 backward and 1 right, 2 backward and 1 left, 1 forward and 2 left, etc..

A "Knight's tour" is a sequence of moves that the knight takes on a chessboard in such a way that it reaches every square on the chessboard. In other words, if you view the squares on the chessboard as the vertices of the graph, a Knight's Tour is a Hamiltonian path. A "Reentrant Knight's Tour" is a knight's tour that is a Hamiltonian circuit, rather than just a path, so that it returns to the square from which it started.

(a) Find a Knight's Tour on a 3×4 chessboard (3 rows and 4 columns).

(b) Show that there is no Reentrant Knight's Tour on a 3×4 chessboard. (Hint: Start with those squares on the board for which a knight would have only two moves. The edges corresponding to those moves must be included in the circuit. Build from there and see what happens.)

(c) Show that there is no Knight's Tour on a 3×3 chessboard.

(d) Is there a Reentrant Knight's Tour on a 4×4 chessboard? If so, show it. If not, explain why not.