

1. Simplify each of the following.

$$(a) \frac{\frac{4x^8}{9}}{\frac{16x^4}{27}}$$

$$(b) \frac{\frac{\frac{1}{2} + \frac{1}{4}}{\frac{1}{2} - \frac{1}{4}}}{\frac{1}{2} - \frac{1}{4}}$$

$$(c) \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$$

$$(d) \frac{\frac{1}{ab}}{\frac{1}{a} + \frac{1}{b}}$$

$$(e) \frac{\frac{1}{x^2 + 7x + 12}}{\frac{1}{x^2 + x - 6}}$$

$$(f) \frac{\frac{x+9}{x^2+6x+9}}{\frac{2}{x^2-9}}$$

$$(g) \frac{1 - \frac{1}{x-5}}{1 - \frac{8}{x+2}}$$

$$(h) \frac{\frac{y}{x} - 1}{\frac{x}{y} + 1}$$

$$(i) \frac{1 - \frac{4}{y^2}}{1 - \frac{1}{y} - \frac{6}{y^2}}$$

2. Solve the following equations.

$$(a) \frac{x}{5} - \frac{2}{5} = \frac{3x}{10}$$

$$(b) \frac{7}{x} = \frac{8}{3}$$

$$(c) \frac{1}{y} - \frac{2}{3} = \frac{1}{4}$$

$$(d) \frac{10}{a-8} = \frac{6}{a-2}$$

$$(e) 1 - \frac{1}{x} = \frac{6}{x^2}$$

$$(f) \frac{5}{x+1} = \frac{3}{x-1}$$

$$(g) \frac{6}{x-3} = \frac{3}{5}$$

$$(h) x = \frac{12}{x+1}$$

$$(i) \frac{x}{2} + \frac{x+4}{x+3} = \frac{1}{x+3}$$

$$(j) \frac{x}{2} + \frac{4}{2x+6} = \frac{1}{2}$$

$$(k) \frac{x}{5} = \frac{8}{5x-15} + \frac{1}{5}$$

$$(l) \frac{x}{2} - \frac{x+5}{x+2} = \frac{3x}{2x+4}$$

$$(m) \frac{9}{x+2} - \frac{5}{x-2} = \frac{4}{x^2-4}$$

3. Solve $x = \frac{3y-2}{4y-1}$ for y .

4. Convert 18 feet per minute to yards per hour.

5. The sum of a number and its reciprocal is $\frac{25}{12}$. Find the numbers.

6. When the same number is added to the numerator and the denominator of the fraction $\frac{7}{9}$, the result is $\frac{5}{6}$. Find the number.

7. A boat travels 48 miles down a river in the same amount of time it takes it to travel 32 miles up the same river. If the speed of the river is 2 miles per hour, what is the speed of the boat in still water?

8. One number is three times another number. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

9. An airplane flying with the wind can travel 840 miles in the same amount of time that it can travel 630 miles flying against the wind. If the speed of the wind is 30 miles per hour, what is the speed of the plane in still air?

10. Evaluate or simplify the expression. Assume that all variables are positive.

$$(a) \sqrt{121}$$

$$(b) \sqrt{64}$$

$$(c) \sqrt[3]{27}$$

$$(d) \sqrt[4]{16}$$

$$(e) \sqrt[5]{-32}$$

$$(f) \sqrt[4]{-625}$$

$$(g) \sqrt[3]{125}$$

$$(h) \sqrt[3]{216}$$

$$(i) \sqrt[3]{.125}$$

$$(j) \sqrt[4]{.0625}$$

$$(k) \sqrt{10000}$$

$$(l) \sqrt{250000}$$

$$(m) 36^{\frac{1}{2}}$$

$$(n) -9^{\frac{1}{2}}$$

$$(o) (-16)^{\frac{1}{2}}$$

$$(p) (-27)^{\frac{1}{3}}$$

$$(q) (-27)^{\frac{2}{3}}$$

$$(r) 81^{-\frac{1}{4}}$$

$$(s) (-64)^{-\frac{2}{3}}$$

$$(t) \left(\frac{64}{25}\right)^{-\frac{1}{2}}$$

$$(u) 16^{\frac{3}{4}}$$

$$(v) 4^{-\frac{3}{2}}$$

$$(w) 49^{-\frac{3}{2}} - 1^{\frac{4}{5}}$$

$$(x) \sqrt[5]{54}$$

$$(y) \sqrt{\frac{1}{5}}$$

$$(z) \sqrt{5}\sqrt{10}$$

$$(aa) \sqrt{3}(2 - \sqrt{3})$$

$$(ab) \frac{\sqrt{12}}{\sqrt{3}}$$

$$(ac) \frac{5}{8}\sqrt{3} + \frac{2}{5}\sqrt{3}$$

$$(ad) \sqrt{50} + 3\sqrt{8}$$

$$(ae) (2\sqrt{8})(3\sqrt{6})$$

$$(af) (2 + \sqrt{6})^2$$

$$(ag) \frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$(ah) \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$$

$$(ai) \sqrt{49a^4y^2}$$

$$(aj) \sqrt{81x^6y^{92}}$$

$$(ak) \sqrt[3]{-27x^{15}y^{12}}$$

$$(al) \sqrt[3]{\frac{8x^3}{125y^9z^6}}$$

$$(am) x^{\frac{3}{5}}x^{\frac{2}{5}}$$

$$(an) \left(x^{\frac{2}{3}}\right)^{\frac{1}{2}}$$

$$(ao) \frac{x^{\frac{2}{7}}}{x^{\frac{5}{7}}}$$

$$(ap) \frac{\left(y^{\frac{5}{3}}\right)^{\frac{3}{4}}}{\left(y^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

$$(aq) \left(\frac{a^{-\frac{1}{5}}}{b^{\frac{1}{3}}}\right)^{15}$$

$$(ar) \frac{4}{x^{\frac{2}{3}}} - x^{\frac{1}{3}}$$

$$(as) \left(2x^{\frac{2}{3}} - 5\right)^2$$

$$(at) \sqrt{16x^3}$$

$$(au) \sqrt{72x^5y^6}$$

$$(av) 3\sqrt[3]{250x^3y^6}$$

$$(aw) \sqrt[3]{\frac{5}{8x^2}}$$

$$(ax) \sqrt[4]{\frac{5x}{27yz^2}}$$

$$(ay) \sqrt[4]{\frac{2x}{8y}}$$

$$(az) 10\sqrt{x} - \sqrt{x}$$

$$(ba) \frac{36x^{\frac{4}{3}} + 18x^{\frac{7}{3}}}{9x^{\frac{2}{3}}}$$

$$(bb) (3\sqrt{x} - 5)^2$$

$$(bc) \frac{x-3}{\sqrt{x} - \sqrt{3}}$$

$$(bd) \frac{\sqrt{6} - \sqrt{x}}{\sqrt{6} + \sqrt{x}}$$

$$(be) 25^{\frac{1}{2}} - 100^{\frac{1}{2}}$$

$$(bf) 25^{-\frac{1}{2}} - 100^{-\frac{1}{2}}$$

$$(bg) 18\sqrt{11} + 9\sqrt{11}$$

$$(bh) \frac{1}{3}\sqrt{27} + \frac{3}{4}\sqrt{48}$$

$$(bi) y^{\frac{2}{3}}\left(y^{\frac{1}{3}} - y^{-\frac{1}{3}}\right)$$

$$(bj) 5x\sqrt{2x} + 3\sqrt{8x^3}$$

$$(bk) \sqrt{147} - \sqrt{192} + \sqrt{75}$$

$$(bl) \sqrt{12} - \sqrt{\frac{4}{3}} + \sqrt{\frac{3}{4}}$$

$$(bm) \sqrt{48} + \sqrt{\frac{3}{16}} - \sqrt{\frac{16}{3}}$$

$$(bn) (\sqrt{7} + 3)(2\sqrt{7} - 5)$$

$$(bo) 5a^{\frac{2}{3}}b^{\frac{3}{4}}\left(ab + a^{\frac{1}{3}}b^{\frac{1}{4}}\right)$$

$$(bp) \left(a^{\frac{3}{4}} + b^{\frac{1}{2}}\right)\left(a^{\frac{3}{4}} - b^{\frac{1}{2}}\right)$$

$$(bq) \frac{44a^{\frac{1}{3}}b^{\frac{7}{4}} - 55a^{-\frac{2}{3}}b^{\frac{11}{4}}}{11a^{-\frac{2}{3}}b^{\frac{7}{4}}}$$

$$(br) 8xy\sqrt{2x} - 6xy\sqrt{2x}$$

- (bs) $8x\sqrt[3]{y} - 4\sqrt[3]{x^3y} + x\sqrt[3]{y}$
 (bt) $12\sqrt{27x^3y^2} + 8xy\sqrt{12x}$
 (bu) $(5\sqrt{6} - \sqrt{2})(2\sqrt{6} + \sqrt{2})$
 (bv) $(\sqrt{5} - \sqrt{18})(6\sqrt{5} + \sqrt{18})$
 (bw) $(\sqrt{5} - 6\sqrt{3})(2\sqrt{5} - 3\sqrt{3})$
 (bx) $9\sqrt{5x^3y^2} - 4x\sqrt{20xy^2} + 7xy\sqrt{5x}$

11. Simplify each of the following, but do *not* assume that the variables are positive.

- (a) $\frac{\sqrt{64x^8y^6}}{2\sqrt{16}}$ (c) $\sqrt[4]{64x^7y^8}$ (e) $\sqrt{27x^3 - 3x^2}$
 (b) $\sqrt[3]{40a^3b^6c^8}$ (d) $\sqrt{x^2 + 4x + 4}$ (f) $\sqrt{\frac{60x^4y}{16z^3}}$

12. Factor each of the following (as if they were quadratics).

- (a) $16y^{\frac{2}{3}} - 9$ (c) $2y^{\frac{1}{2}} + 5y^{\frac{1}{4}} - 12$
 (b) $x^{\frac{2}{3}} - 4x^{\frac{1}{3}} - 12$

13. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.
 Find the radius of the sphere if the volume is 8π cubic feet.

14. A triangle is equilateral if it has three equal sides. Find the ratio of the height of an equilateral triangle to one of its sides.

15. Solve each of the following equations.

- (a) $\sqrt{1-x} = 5$ (f) $x - 2 = \sqrt{3x+4}$
 (b) $3 + \sqrt{x+8} = 0$ (g) $\sqrt{x+4} = x+4$
 (c) $\sqrt[3]{5x-9} = 2$ (h) $\sqrt{2x+19} = x+2$
 (d) $\sqrt{2a+7} = 7$ (i) $\sqrt{5y-1} = \sqrt{5y-1}$
 (e) $\sqrt[3]{5n-4} = \sqrt[3]{2-7n}$ (j) $\sqrt{x-8} = \sqrt{x-4} + 2$

16. Solve the formula $t = \sqrt{\frac{2h-40t}{g}}$ for g .

17. Solve the formula $T = 2\pi\sqrt{\frac{L}{g}}$ for L .

ANSWERS

1. (a) $\frac{3x^4}{4}$ (f) $\frac{(x+9)(x-3)}{2(x+3)}$ (ah) $\frac{4-x}{x^{\frac{2}{3}}}$ (bh) $4\sqrt{3}$
 (b) 3 (g) $\frac{x+2}{x-5}$ (ai) $4x^{\frac{4}{3}} - 20x^{\frac{2}{3}} + 25$ (bi) $y - y^{\frac{1}{3}}$
 (c) $\frac{x+1}{x-1}$ (h) $\frac{y(y-x)}{x(x+y)}$ (at) $4x\sqrt{x}$ (bj) $11x\sqrt{2x}$
 (d) $\frac{1}{b+a}$ (i) $\frac{y-2}{y-3}$ (au) $6x^2y^3\sqrt{2x}$ (bk) $4\sqrt{3}$
 (e) $\frac{x-2}{x+4}$ (j) $x = -1$ (av) $15xy^2\sqrt[3]{2}$ (bl) $\frac{11\sqrt{3}}{6}$
 2. (a) $x = -4$ (g) $x = 13$ (aw) $\frac{\sqrt[3]{5x}}{2x}$ (bm) $\frac{35\sqrt{3}}{12}$
 (b) $x = \frac{21}{8}$ (h) $x = -4$ or $x = 3$ (ax) $\frac{\sqrt[4]{15xy^3z^2}}{3yz}$ (bn) $\sqrt{7} - 1$
 (c) $y = \frac{12}{11}$ (i) $x = -2$ (ay) $\frac{\sqrt[4]{4xy^3}}{2y}$ (bo) $5a^{\frac{5}{3}}b^{\frac{7}{4}} + 5ab$
 (d) $a = -7$ (j) $x = -1$ (az) $9\sqrt{x}$ (bp) $a^{\frac{3}{2}} - b$
 (e) $x = 3$ or $x = -2$ (k) $x = 5$ or $x = -1$ (ba) $4x^{\frac{2}{3}} + 2x^{\frac{5}{3}}$ (bq) $4a - 5b$
 (f) $x = 4$ (l) $x = 5$ (bb) $9x - 30\sqrt{x} + 25$ (br) $2xy\sqrt{2x}$
 3. $y = \frac{x-2}{4x-3}$ (m) $x = 8$ (bc) $\sqrt{x} + \sqrt{3}$ (bs) $5x\sqrt[3]{y}$
 4. 360 yards per hour (bd) $\frac{6 - 2\sqrt{6x} + x}{6 - x}$ (bt) $52xy\sqrt{3x}$
 5. The numbers are $\frac{4}{3}$ and $\frac{3}{4}$. (be) -5 (bu) $58 + 6\sqrt{3}$
 6. The number is 3. (bf) $\frac{1}{10}$ (bv) $12 - 15\sqrt{10}$
 7. The boat's speed is 10 mph. (bg) $27\sqrt{11}$ (bw) $64 - 15\sqrt{15}$
 8. The numbers are 4 and 12. (d) $|x^4y^3|$ (d) $|x+2|$
 9. The speed of the plane is 210 mph. (e) $2ab^2c^2\sqrt[3]{5c^2}$ (e) $|x|\sqrt{27x-3}$
 10. (a) 11 (q) 9 (ad) $11\sqrt{2}$ (f) $\frac{x^2\sqrt{15yz}}{2z^2}$
 (b) 8 (r) $\frac{1}{3}$ (ae) $24\sqrt{3}$ (c) $\frac{2y^{\frac{1}{5}} - 3}{(2y^{\frac{1}{4}} - 3)(y^{\frac{1}{4}} + 4)}$
 (c) 3 (s) $\frac{1}{16}$ (af) $10 + 4\sqrt{6}$ 11. (a) $|x^4y^3|$ (d) $|x+2|$
 (d) 2 (t) $\frac{5}{8}$ (ag) $\sqrt{6} - 2$ (e) $|x|\sqrt{27x-3}$
 (e) -2 (u) 8 (ah) $-5 - 2\sqrt{6}$ (f) $\frac{x^2\sqrt{15yz}}{2z^2}$
 (f) does not exist (v) $\frac{1}{8}$ (ai) $7a^2y$ 12. (a) $(4y^{\frac{1}{5}} - 3)(4y^{\frac{1}{5}} + 3)$ (c) $(2y^{\frac{1}{4}} - 3)(y^{\frac{1}{4}} + 4)$
 (g) 5 (w) $-\frac{342}{343}$ (aj) $9x^3y^{46}$ 13. The radius is $\sqrt[3]{6}$ feet.
 (h) 6 (x) $3\sqrt[3]{2}$ (ak) $-3x^5y^4$ 14. ratio = $\frac{\sqrt{3}}{2}$
 (i) $\frac{1}{2}$ (y) $\frac{\sqrt{5}}{5}$ (al) $\frac{2x}{5y^3z^2}$ 15. (a) $x = -24$ (f) $x = 7$
 (j) $\frac{1}{2}$ (z) $5\sqrt{2}$ (am) x (g) $x = -4, x = -3$
 (k) 100 (aa) $2\sqrt{3} - 3$ (an) $x^{\frac{1}{3}}$ (h) $x = 3$
 (l) 500 (ab) 2 (ao) $\frac{1}{x^{\frac{3}{7}}}$ (i) $y = \frac{1}{5}$
 (m) 6 (ac) $\frac{41\sqrt{3}}{40}$ (ap) y (j) no solution
 (n) -3 (aq) $\frac{1}{a^3b^5}$
 (o) does not exist
 (p) -3