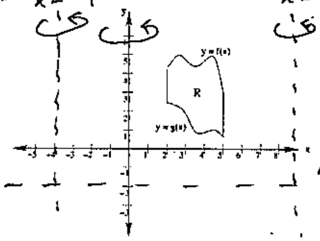
81 − 90 = 9
71 − 80 ⇒ 8
61 − 70 ⇒ 7
−60 ⇒ 6

8

Math 262 Calculus II Lab 4 Solids of Revolution Name: KEY

1. Let R be the region bounded by the graphs of $y = f(x)$, $y = g(x)$, $x = 2$, and $x = 5$ (see graph below).

$x = -4$ $\quad$ $x = 9$ $\quad$ $y = 8$



$y = -2$

+2

Set up an integral (you do **not** need to evaluate it) that can be used to find each of the following:

(a) The area of R

$$A = \int_2^5 [f(x) - g(x)]\,dx$$

(b) The volume of the solid generated if R is revolved about the x-axis.

$$V = \int_2^5 \pi[r_o^2 - r_i^2]\,dx = \pi\int_2^5 [(f(x))^2 - (g(x))^2]\,dx$$

(c) The volume of the solid generated if R is revolved about the y-axis.

$$V = 2\pi\int_2^5 rh\,dx = 2\pi\int_2^5 x(f(x) - g(x))\,dx$$

4 each

(d) The volume of the solid generated if R is revolved about $x = 9$.

$$V = 2\pi\int_2^5 (9-x)(f(x) - g(x))\,dx$$

(e) The volume of the solid generated if R is revolved about $x = -4$.

$$V = 2\pi\int_2^5 (4+x)(f(x) - g(x))\,dx$$

(f) The volume of the solid generated if R is revolved about $y = 8$.

$$V = \pi\int_2^5 [(8-g(x))^2 - (8-f(x))^2]\,dx$$

(g) The volume of the solid generated if R is revolved about $y = -2$.

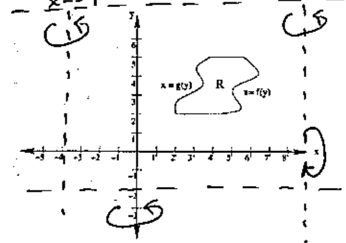
$$V = \pi\int_2^5 [(f(x)+2)^2 - (g(x)+2)^2]\,dx$$

28

Math 262 Calculus II Lab 4 Solids of Revolution Name: KEY

2. Let R be the region bounded by the graphs of $x = f(y)$, $x = g(y)$, $y = 2$, and $y = 5$ (see graph below).

$x = -4$ $\quad$ $x = 9$ $\quad$ $y = 8$



$y = -2$

Set up an integral (you do **not** need to evaluate it) that can be used to find each of the following:

(a) The area of R

$$A = \int_2^5 (f(y) - g(y))\,dy$$

(b) The volume of the solid generated if R is revolved about the x-axis.

$$V = 2\pi\int_2^5 y(f(y) - g(y))\,dy$$

(c) The volume of the solid generated if R is revolved about the y-axis.

$$V = \pi\int_2^5 [(f(y))^2 - (g(y))^2]\,dy$$

(d) The volume of the solid generated if R is revolved about $x = 9$.

$$V = \pi\int_2^5 [(9-g(y))^2 - (9-f(y))^2]\,dy$$

(e) The volume of the solid generated if R is revolved about $x = -4$.

$$V = \pi\int_2^5 [(4+f(y))^2 - (4+g(y))^2]\,dy$$

(f) The volume of the solid generated if R is revolved about $y = 8$.

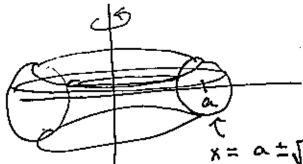
$$V = 2\pi\int_2^5 (8-y)(f(y) - g(y))\,dy$$

(g) The volume of the solid generated if R is revolved about $y = -2$.

$$V = 2\pi\int_2^5 (2+y)(f(y) - g(y))\,dy$$

$\frac{28}{56}$

3. A doughnut-shaped solid is called a **torus**. Use the WASHER METHOD to calculate the volume of the torus obtained by rotating the region inside the circle with equation $(x-a)^2+y^2=b^2$ around the y-axis (assume that $a > b$). Hint: Evaluate the integral by interpreting it as the area of a circle.



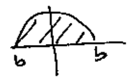
$x = a \pm \sqrt{b^2-y^2}$ (+2)

$$V = \pi \int_{-b}^{b} \left[(a+\sqrt{b^2-y^2})^2 - (a-\sqrt{b^2-y^2})^2 \right] dy \quad (+3)$$

$$= \pi \int_{-b}^{b} \left[a^2 + 2a\sqrt{b^2-y^2} + b^2 - y^2 - a^2 + 2a\sqrt{b^2-y^2} - b^2 + y^2 \right] dy$$

$$= \pi \int_{-b}^{b} 4a\sqrt{b^2-y^2}\, dy \quad (+2)$$

$$= 4a\pi \int_{-b}^{b} \sqrt{b^2-y^2}\, dy$$

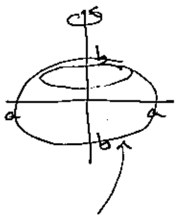


$$= 4a\pi\left(\tfrac{1}{2}\pi b^2\right)$$

$$= 2\pi^2 a b^2 \quad (+3)$$

10 pts

4. Let R be the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Show that the solid obtained by rotating R about the y-axis (called an **ellipsoid**) has volume $\frac{4}{3}\pi a^2 b$.



$x = \pm a\sqrt{1-\frac{y^2}{b^2}}$

$= \pm \frac{a}{b}\sqrt{b^2-y^2}$ (+2)

$$V = \pi \int_{-b}^{b} \left(\frac{a}{b}\sqrt{b^2-y^2}\right)^2 dy \quad (+3)$$

$$= \pi \frac{a^2}{b^2} \int_{-b}^{b} (b^2-y^2)\, dy \quad (+1)$$

$$= \frac{a^2}{b^2}\pi \left[b^2 y - \tfrac{1}{3}y^3 \right|_{-b}^{b} \quad (+2)$$

$$= \frac{a^2}{b^2}\pi \left[b^3 - \tfrac{1}{3}b^3 - (-b^3 + \tfrac{1}{3}b^3) \right]$$

$$= \frac{a^2}{b^2}\pi \left[\tfrac{4}{3} b^3 \right]$$

$$= \tfrac{4}{3}\pi a^2 b \quad (+2)$$

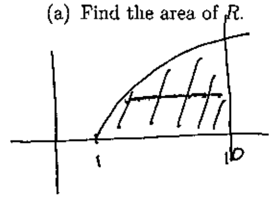
10 pts

$\frac{20}{76}$

5. (From the 200? AP Calculus AB exam.) Let R be the region enclosed by the graph of $y=\sqrt{x-1}$, the vertical line $x=10$, and the x-axis.

$x = y^2+1$

(a) Find the area of R.



$$A = \int_1^{10} (\sqrt{x-1})\, dx \quad (+2)$$

$$= \tfrac{2}{3}(x-1)^{3/2} \Big|_1^{10} \quad (+2)$$

$$= \tfrac{2}{3}(9 \cdot 3 - 0)$$

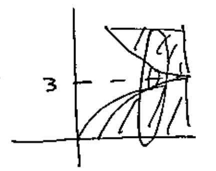
$$= 18 \quad (+2)$$

$$\int_0^3 (10 - y^2 - 1)\, dy$$

$$= 9y - \tfrac{1}{3}y^3 \Big|_0^3 = 27 - 9 = 18$$

6 pts

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y=3$.



$$V = \pi \int_1^{10} \left(3^2 - (3-\sqrt{x-1})^2 \right) dx \quad (+3)$$

$$= \pi \int_1^{10} (9 - 9 + 6\sqrt{x-1} - x + 1)\, dx \quad (+1)$$

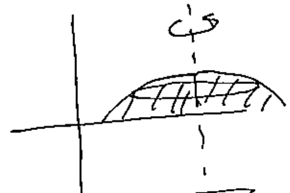
$$= \pi \left[6 \cdot \tfrac{2}{3}(x-1)^{3/2} - \tfrac{1}{2}x^2 + x \right|_1^{10} \quad (+2)$$

$$= \pi \left[4 \cdot 9 \cdot 3 - 50 + 10 - 0 + \tfrac{1}{2} - 1 \right]$$

$$= 67.5\pi = \frac{135\pi}{2} \quad (+2)$$

8 pts

(c) Find the volume of the solid generated when R is revolved about the vertical line $x=10$.



$y = \sqrt{x-1}$

$x = y^2+1$

$$V = \pi \int_0^3 (10 - y^2 - 1)^2\, dy \quad (+3)$$

$$= \pi \int_0^3 (81 - 18y^2 + y^4)\, dy \quad (+1)$$

$$= \pi \left[81y - 6y^3 + \tfrac{1}{5}y^5 \right|_0^3 \quad (+2)$$

$$= \pi \left[243 - 162 + \tfrac{243}{5} \right] = \frac{648\pi}{5} \quad (+2)$$

8 pts

$\frac{22}{98}$