This sheet does not have room to work on it, so please submit your completed assignment on your own paper.

1. (3 points) Find an exact differential equation, in differential form, such that the general solution (given in implicit form) is: $3x^2 y\cos\left(x^2+1\right) = C$.

2. Consider the differential equation: $\left(3y - \dfrac{y}{(x+2)^2} + 15e^{3x}\right)dx + \left(3x + \dfrac{1}{x+2}\right)dy = 0.$

   (a) (1 point) Show that the differential equation is exact.

   (b) (4 points) Since the differential equation is exact, $M(x,y) = \dfrac{\partial f}{\partial x}$ and $N(x,y) = \dfrac{\partial f}{\partial y}$. Find an expression for $f(x,y)$.

   (c) (1 point) Find an implicit expression that gives the solution $y = y(x)$ of the differential equation.

   (d) (1 point) Solve your answer from part (2c) above for y to find an explicit solution of the DE.

   (e) (1 point) Give the interval for your solution if the initial condition were to be given at $x = -5$.

3. Consider the differential equation: $\left(6xy - 7\cos(y) + 15\sec^2(x)\right)dx + \left(3x^2 + 7x\sin(y)\right)dy = 0$

   (a) (1 point) Show that the differential equation is exact.

   (b) (4 points) Solve the differential equation by integrating with respect to x first.

   (c) (4 points) Solve the differential equation by integrating with respect to y first.

4. (6 points) Solve the initial value problem:
   $\left(\dfrac{1}{1+y^2} + \cos(x) - 2xy\right)\dfrac{dy}{dx} = y\left(y + \sin(x)\right)$ with $y(0) = 1$.

5. Sometimes it is possible to find an integrating factor that will turn a differential equation, written in differential form, from a differential equation that is not exact to a differential equation that is exact. This problem illustrates that process.

   Consider the differential equation: $\left(x^2 + y^2 + 4\right)dx + xydy = 0.$

   (a) (1 point) Show that the differential equation as given is not exact.

   (b) (2 points) Find $\dfrac{M_y - N_x}{N}$ and simplify. $\left(\text{Recall that } M_y \text{ is another notation for } \dfrac{\partial M}{\partial y}.\right)$ Is the result a function of x alone?

   (c) (2 points) Find $\dfrac{N_x - M_y}{M}$ and simplify. Is the result a function of y alone?

   (d) (2 points) You should have answered "yes" to part (5b) above. Find $\mu(x) = e^{\int \frac{M_y - N_x}{N}dx}$.

   (e) (1 point) Multiply both sides of the original differential equation by your $\mu(x)$.

   (f) (1 point) Show that this new form of the differential equation is now exact.

   (g) (5 points) Solve the differential equation using the exact form.

   Note: If you can answer "yes" to part (5b), then $\mu(x) = e^{\int \frac{M_y - N_x}{N}dx}$.
   If you can answer "yes" to part (5c), then $\mu(y) = e^{\int \frac{N_x - M_y}{M}dy}$.
   If both answers would be "no", then an integrating factor approach does not work.