This miniproject asks you to justify some standard rules of inference that will later help you in proving more complicated and specific propositions.

Give formal two-column proofs of each of the following.

(a) Universal Modus Tollens

$$\begin{array}{r} \forall x\,(P(x) \to Q(x)) \\ \neg Q(a) \\ \hline \neg P(a) \end{array}$$

(b) Universal Transitivity

$$\begin{array}{r} \forall x\,(P(x) \to Q(x)) \\ \forall x\,(Q(x) \to R(x)) \\ \hline \forall x\,(P(x) \to R(x)) \end{array}$$

(c)

$$\begin{array}{r} \forall x\,(P(x) \to (Q(x) \wedge S(x))) \\ \forall x\,(P(x) \wedge R(x)) \\ \hline \forall x\,(R(x) \wedge S(x)) \end{array}$$