

Do all problems. Give exact answers unless otherwise directed.

1. Consider the function
- $f(x) = 3x^2 - 30x + 97$
- .

(a) Write the function in standard form.

$$3(x^2 - 10x + 25) + 97 - 75$$

$$f(x) = 3(x-5)^2 + 22$$

(b) State the vertex.

$$(5, 22)$$

(c) State the maximum or minimum, and state whether it is a maximum or a minimum.

min ~~value~~ is 22

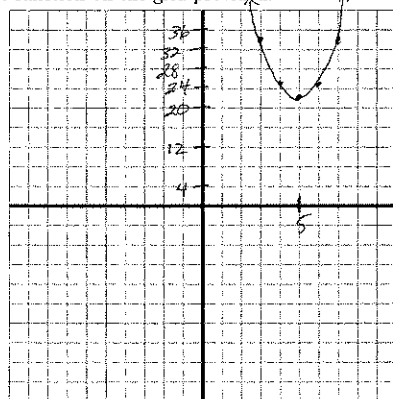
(d) Give the axis.

$$x = 5$$

(e) Using an appropriate and labeled scale, graph the function on the grid provided.

$$f(4) = 3(4) + 22 = 25$$

$$f(3) = 3(4) + 22 = 34$$



2. Consider
- $f(x) = 2x^4 - x^3 - 15x^2 + 8x + 20$

(a) List all of the possible rational roots (you don't have to check them yet).

$$p: 1, 2, 4, 5, 10, 20$$

$$q: 1, 2$$

$$\text{Possible: } \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20, \pm \frac{1}{2}, \pm \frac{5}{2}$$

(b) Using Descartes' Rule of Signs, what are the possible numbers of positive real roots?

2 sign changes,

so 2 or 0 positive roots

(c) Find all of the roots of the polynomial, including their multiplicities.

$$\begin{array}{r|rrrrrr} 2 & 2 & -1 & -15 & 8 & 20 \\ & & 2 & 1 & -14 & -6 \\ \hline & 2 & 1 & -14 & -6 & 14 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 2 & -1 & -15 & 8 & 20 \\ & & 4 & 6 & -18 & -20 \\ \hline & 2 & 3 & -9 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 2 & 3 & -9 & -10 \\ & & 4 & 14 & 10 \\ \hline & 2 & 7 & 5 & 0 \end{array}$$

$$(2x^2 + 7x + 5) = 0$$

$$(2x+5)(x+1) = 0$$

$$x = 2 \text{ (mult. 2)}, x = -\frac{5}{2}, x = -1$$

(d) State the factored form of the polynomial.

$$(2x+5)(x+1)(x-2)^2$$

Do all problems. Give exact answers unless otherwise directed.

1. For each of the following problems, do the following.

(i) State the inequality that you would use to find the domain of the function.

(ii) Find the domain, giving your answer in interval notation.

(a)  $f(x) = (x^2 + 7x + 10)\sqrt{3x - 5}$

(b)  $f(x) = \log_4(9 - 15x)$

$$3x - 5 \geq 0$$

$$x \geq \frac{5}{3}$$

$$\left[ \frac{5}{3}, \infty \right)$$

$$9 - 15x > 0$$

$$-15x > -9$$

$$x < \frac{9}{15} = \frac{3}{5}$$

$$(-\infty, \frac{3}{5})$$

2. Solve the following non-linear inequality using a table of signs.

$$\frac{(7x+1)(x-3)^2}{2x-19} \geq 0$$

$$x = -\frac{1}{7}, 3, \frac{19}{2}$$

	$-\frac{1}{7}$	3	$\frac{19}{2}$	
Test	-1	0	5	100
$7x+1$	-	+	+	+
$(x-3)^2$	+	+	+	+
$2x-19$	-	-	-	+
	(+)	-	-	(+)

$$(-\infty, -\frac{1}{7}] \cup (\frac{19}{2}, \infty)$$

3. Solve the system of equations given below.

$$\begin{cases} 7x + 5y = 12 \\ -2x + 10y = 15 \end{cases}$$

$$\begin{cases} -14x - 10y = -24 \\ -2x + 10y = 15 \end{cases}$$

$$\begin{cases} -16x = -9 \\ -2x + 10y = 15 \end{cases}$$

$$-16x = -9$$

$$x = \frac{9}{16} \Rightarrow y = \frac{12 - 7(\frac{9}{16})}{5} = \frac{192 - 63}{16 \cdot 5} = \frac{129}{80} \quad (\frac{9}{16}, \frac{129}{80})$$

$$y = \frac{12 - 7(\frac{9}{16})}{5} = \frac{192 - 63}{16 \cdot 5} = \frac{129}{80}$$

4. Use synthetic division to do the division below. Make sure that you remember to state the answer.

$$\frac{7x^5 + 15x^4 + 5x^3 + 4x^2 + 6x + 25}{x+2}$$

$$\begin{array}{r|rrrrrr} -2 & 7 & 15 & 5 & 4 & 6 & 25 \\ & & -14 & -2 & -6 & 4 & -20 \\ \hline & 7 & 1 & 3 & -2 & 10 & 5 \end{array}$$

$$7x^4 + x^3 + 3x^2 - 2x + 10 + \frac{5}{x+2}$$

5. Use synthetic division and the Remainder Theorem to find the following.

(a)  $f(4)$  if  $f(x) = x^4 - 10x^3 + 25x^2 - 21$

(b)  $f(1+i)$  if  $f(x) = -x^2 + x + 6$

$$\begin{array}{r|rrrrr} 4 & 1 & -10 & 25 & 0 & -21 \\ & & 4 & -24 & 4 & 16 \\ \hline & 1 & -6 & 1 & 4 & -5 \end{array}$$

$$f(4) = -5$$

$$\begin{array}{r|rrrr} 1+i & -1 & 1 & 6 & (-1) \\ & & -1-i & -i-1 & -i-2 \\ \hline & -1 & -i & -i-1 & -i-2 \end{array}$$

$$f(1+i) = 5-i$$

6. Find a polynomial with real coefficients, leading coefficient 12, and zeros  $x = 5$ ,  $x = -2$  (multiplicity 5),  $x = -12$  (multiplicity 2),  $x = 4i$ , and  $x = 0$ . Leave your answer in factored form.

$$P(x) = 12x(x-5)(x+2)^5(x+12)^2(x-4i)(x+4i)$$