Math 335 — Test 4 Review Sheet

The exam covers Sections 4.9, 5.1–5.5.

1. Distributions:

   (a) Multinomial (discrete)

   - Context: Generalization of binomial when there are more than two possible outcomes.
   - Must be independent trials
   - $f(x_1, x_2, \ldots, x_k)$
     $= \dfrac{n!}{x_1!x_2!\cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$
     where $x_1 + x_2 + \cdots + x_k = n$ and $p_1 + p_2 + \cdots + p_k = 1$

   (b) Standard normal (continuous)

   - density $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$
   - distribution $F(z) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{\frac{-t^2}{2}} dt$
   - $\mu = 0$, $\sigma = 1$
   - Table 3
     - Be able to use symmetry properties to find probabilities not given directly by Table 3.
   - Bell-shaped curve, centered at $\mu = 0$
   - Be able to find z_α, which is the z-score for which the upper tail has probability α.

   (c) Normal (continuous)

   - density $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
   - distribution
     $F(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-(t-\mu)^2}{2\sigma^2}} dt$
   - mean is μ, standard deviation is σ
   - Be able to convert to a z-score
     $z = \dfrac{x-\mu}{\sigma}$
   - Bell-shaped curve, centered at μ

   (d) Uniform (continuous)

   - $f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \text{if } \alpha < x < \beta \\ 0 & \text{elsewhere} \end{cases}$
   - $\mu = \dfrac{\alpha+\beta}{2}$
   - $\sigma = \dfrac{\beta-\alpha}{\sqrt{12}}$

2. Continuous versus discrete probability distributions

3. For continuous distributions:

   (a) f is probability density function, does *not* give the probabilities

   (b) F is the cumulative probability distribution, does give the probabilities

   (c) Need $\int_{-\infty}^{\infty} f(x)dx = 1$

   (d) $F(x) = \int_{-\infty}^{x} f(t)dt$

   (e) $P(X \le a) = \int_{-\infty}^{a} f(t)dt = F(a)$

   (f) $P(a \le X \le b) = \int_{a}^{b} f(t)dt = F(b) - F(a)$

   (g) Mean $\mu = \int_{-\infty}^{\infty} xf(x)dx$

   (h) $\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)dx$

   (i) Variance $\sigma^2 = \mu_2' - \mu^2$ or
   $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

4. Normal approximation to the Binomial

   - Rule of thumb is:
     $np > 15$ **and** $n(1-p) > 15$
   - $\mu = np$
   - $\sigma = \sqrt{np(1-p)}$