You must do a minimum of eight portfolio proofs, with the distribution indicated below. The portfolio proofs will be worth 30% of your overall course grade for Math 311. You may do more than eight proofs in order to boost your grade, but only after you have eight graded portfolio proofs first.

A proof will be graded only if it is both substantially correct and well-written. A "proof" that has a large error in it will be returned to you ungraded to be re-worked and re-submitted. Likewise, a "proof" that is poorly written will be returned ungraded to be re-written. Poorly written proofs include those that have only algebra and no words, poor grammar, a clear lack of a consistent audience, frequent misspelled words, random capitalization, incomplete or incoherent sentences, are illegibly hand-written, etc.

Each proof will be graded on a 10-point scale, and the maximum you may receive for portfolio proofs is 80 points.

Note that proofs that have small errors in either the Mathematics or the English may still receive poor grades. A proof that is graded may still be re-submitted with corrections to have the grade improved.

When re-submitting proofs, you must include the previous submission(s), stapled behind your new submission.

Note: This will be filled in with the exact problems as the term progresses. Updates will be posted to my webpage.

1. You must do one of the following two induction proofs.
   (a) For all integers n that are greater than 6, we have $3^n < n!$.
   (b) $\displaystyle\sum_{k=1}^{n} \frac{2}{(k+3)(k+5)} = \frac{9}{20} - \frac{2n+9}{(n+4)(n+5)}$
2. You must do one of the following two set-inclusion proofs.
   (a) Prove using an element argument that $A - (A \cap B \cap C) = (A - B) \cup (A - C)$.
   (b) Prove using an element argument that $(C - B) \cup (A \cap B \cap C) = (A \cap C) \cup (C - B)$.
3. You must do one of the following two logic proofs, using a formal two-column proof.
   (a) All Dragons hoard treasures. There is a student who writes proofs. All students are Dragons. Therefore, there is a student who hoards treasures.
   (b) Frogs that are not green are poisonous. No poisonous creatures live in Minnesota. There is a frog that lives in Minnesota. Therefore, there is a frog that is green.
4. You must do one of the following two proofs for a Boolean Algebra, using a formal two-column proof and only the standard named properties for a Boolean Algebra (see Table 5).
   (a) $(x+y)\,\overline{x\,y} = x\,\overline{y} + \overline{x}\,y$
   (b) $x\,\overline{y} + (\overline{x}\,y + \overline{x}\,\overline{y}) = \overline{x} + \overline{y}$

5. You must do four of the following eight general proofs. Your proofs should be paragraph-style proofs.

   (a) Prove that $\sqrt{19}$ is irrational.

   (b) Prove: If x and y are rational numbers with $y \neq 0$, then $\dfrac{x}{y}$ is also rational.

   (c) Prove: The sum of an even integer and an odd integer is odd.

   (d) Prove: If a divides b and a divides c, then a divides $b - c$.

   (e) Use the definition of a limit to prove: $\lim_{x \to 3} (2x + 9) = 15$.
   Recall the definition of a limit: $\forall \epsilon, \exists \delta > 0$ such that $0 < |x - a| < \delta \longrightarrow |f(x) - L| < \epsilon$.

   (f) Write the numbers $1, 2, 3, \ldots, 2n$ on a blackboard, where n is an odd positive integer. Pick any two of the numbers, say j and k. Write $|j - k|$ on the blackboard and erase both j and k. Continue in this manner until only one integer is written on the board. Prove that this final number must be odd.

   (g) Suppose that five ones and six zeros are arranged around a circle. Between any two equal bits you insert a 0 and between any two unequal bits you insert a 1 to produce eleven new bits. Then you erase the eleven original bits. This procedure is then repeated many times. Show that you can never get eleven zeros. [Hint: Work backward, assuming that you did end up with eleven zeros.]

   (h) Let R be a symmetric relation. Then R^n is symmetric for all positive integers n.