MA 102 Extra Worksheet on Counting Problems Name Key - To be Returned Seat ____

Show your work!!!

1. The board of directors of a corporation has 15 members. In how many ways can an executive committee of 4 members, (President, VP, Treasurer, and Secretary) be chosen?

$${}_{15}P_4 = \frac{15!}{(15-4)!} = 15 \cdot 14 \cdot 13 \cdot 12 = \boxed{32{,}760 \text{ executive committees}}$$

2. How many ways can the same board of directors choose a delegation of 4 members where all the delegates have equal standing?

$${}_{15}C_4 = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4!} = \boxed{1{,}365 \text{ committees}}$$

3. There are 20 singers auditioning for a musical. The director wants to choose two people to sing a duet and everyone auditioning is capable of singing either part. In this situation:

${}_{20}C_2$ would be the number of ways to choose two people among 20 where no parts are assigned

${}_{20}P_2$ would be the number of ways to choose two people among 20 where each is assigned to a different part.

4. How many ways can the director choose a lead singer and a backup singer from the 20 persons auditioning?

$${}_{20}P_2 = \frac{20!}{(20-2)!} = 20 \cdot 19 = \boxed{380 \text{ ways}}$$

5. In the situation where the director is choosing a quintet (5 singers):

${}_{20}P_5$ would be the number of ways to select a group of 5 singers among 20 people with specific parts are assigned

${}_{20}C_5$ would be the number of ways to select a group of singers among 20 people without parts be assigned.

6. There are 117 Division I-A college football teams. How many different TOP 25 rankings lists are possible?

$${}_{117}P_{25} = \frac{117!}{(117-25)!} = \frac{117!}{92!} = \boxed{3.19 \times 10^{50} \text{ rankings}}$$

7. For the same 117 teams, how many ways are there to choose 8 teams for a playoff?

$${}_{117}C_8 = \frac{117!}{(117-8)!\,8!} = \frac{117!}{109!\,8!} = \boxed{6.82 \times 10^{11} \text{ ways}}$$

8. Four men and 4 women line up at the checkout stand in a grocery store. In how many ways can they line up?

$${}_8P_8 = \underset{1^{st}}{8} \cdot \underset{2^{nd}}{7} \cdot \underset{3^{rd}}{6} \cdot \underset{4^{th}}{5} \cdot \underset{5^{th}}{4} \cdot \underset{6^{th}}{3} \cdot \underset{7^{th}}{2} \cdot \underset{8^{th}}{1} = 8! = \boxed{40{,}320 \text{ way}}$$

Key - To be Returned

9. In how many ways can 4 men and 4 women line up if they must alternate woman, man, woman, man, and so on and if a woman must always be first in line?

$_4P_4 \cdot {}_4P_4$ women and men

$\underset{w}{4} \cdot \underset{m}{4} \cdot \underset{w}{3} \cdot \underset{m}{3} \cdot \underset{w}{2} \cdot \underset{m}{2} \cdot \underset{w}{1} \cdot \underset{m}{1} = \boxed{576 \text{ ways}}$

10. The ski club has 35 members (15 females and 20 males).

a. How many ways are there to choose a president, vice-president, and treasurer (no one can serve in two offices at the same time).

$_{35}P_3 = \underset{P}{35} \cdot \underset{VP}{34} \cdot \underset{T}{33} = \boxed{39{,}270 \text{ ways}}$

b. How many different ways can the president, vice-president, and treasurer be chosen if there is the additional requirement that the president must be female?

$\underset{P}{15} \cdot \underset{VP}{34} \cdot \underset{T}{33} = \boxed{16{,}830 \text{ ways}}$

c. How many different ways can these 3 offices be filled if there is a regulation that says the top 3 offices can NOT be held by all men or by all women?

$39{,}270 - (\underbrace{15 \cdot 14 \cdot 13}_{\text{All Women}} + \underbrace{20 \cdot 19 \cdot 18}_{\text{All Males}}) = $

$39{,}270 - (2730 + 6840) = \boxed{29{,}700}$

11. How many 7-digit numbers (i.e, numbers between 1,000,000 and 9,999,999) are even numbers?

$\underset{1-9}{9} \; \underset{0-9}{10} \; \underset{0-9}{10} \; \underset{0-9}{10} \; \underset{0-9}{10} \; \underset{0-9}{10} \; \underset{\text{even}}{5} = \boxed{4{,}500{,}000 \text{ numbers}}$

12. How many 7-digit numbers (see above) are divisible by 5?

$\underset{1-9}{9} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10} \cdot \underset{0-9}{10} \; \underset{\text{divisible by 5 so 0 or 5}}{2} = \boxed{1{,}800{,}000 \text{ numbers}}$

13. A computer password consists of 4 letters (A through Z) followed by a single digit (0 through 9). Assume that the passwords are case sensitive (i.e., uppercase letters are considered different from lowercase letters).

a. how many different passwords are possible? $\underbrace{52 \cdot 52 \cdot 52 \cdot 52}_{\text{A-Z or a-z}} \cdot \underset{0-9}{10} = \boxed{73{,}116{,}160 \text{ words}}$

b. how many different passwords start with Z?

$1 \cdot 52 \; 52 \; 52 \; 10 = \boxed{1{,}406{,}080}$

c. How many different passwords do not start with either z or Z?

$50 \cdot 52 \cdot 52 \cdot 52 \cdot 10 = \boxed{70{,}304{,}000}$

d. How many different passwords have no Z's in them, either uppercase or lowercase?

$50 \; 50 \; 50 \; 50 \; 10 = \boxed{62{,}500{,}000}$

To be Key - Return

14. A set of reference books consists of 8 volumes numbered 1 through 8.

a. In how many ways can the 8 books be arranged on a shelf? Per.

$$_8P_8 \text{ or } \underline{8}\cdot\underline{7}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 8! = \boxed{40{,}320 \text{ ways}}$$

b. In how many ways can the 8 books be arranged on the shelf so that at least 1 book is out of order?

$$40{,}320 - \underset{\substack{\uparrow \\ \text{only a single correct order}}}{1} = \boxed{40{,}319 \text{ ways}}$$

15. A restaurant offers a menu consisting of 3 different appetizers, 2 different soups, 4 different salads, 9 different main courses, and 5 different desserts.

a. A **fixed-price lunch** meal consists of a choice of appetizer, salad, and main course. How many different lunch fixed-price meals are possible?

$$\underset{ap.}{\underline{3}} \cdot \underset{Sal.}{\underline{4}} \cdot \underset{Main}{\underline{9}} = \boxed{108 \text{ lunches}}$$

b. A **fixed-price dinner** meal consists of a choice of appetizer, a choice of soup or salad, a main course, and a dessert. How many different dinner fixed-price meals are possible?

$$\underset{ap}{\underline{3}} \cdot \underset{\text{Soup or Sal}}{\underline{(2+4)}} \cdot \underset{Main}{\underline{9}} \cdot \underset{Des}{\underline{5}} = \boxed{810 \text{ dinners}}$$

c. The **dinner special** consists of a choice of soup or salad or both, and a main course. How many dinner specials are there?

$$\left[\underset{\text{Soup or Salad}}{(2+4)} \overset{\text{or}}{+} \underset{\text{Soup and Sal}}{(2\cdot 4)}\right] \cdot \underset{main}{\underline{9}} = [6+8]\cdot 9 = \underline{14\cdot 9}$$

$$\boxed{126 \text{ dinner specials}}$$

16. As part of a prime-time TV game, the player wins $1,000,000 if the first four cards drawn from a shuffled poker deck are an Ace, a King, a Queen, and a Jack (in any order). How many 4 card combinations of this type are "winning" foursomes?

$$\underset{\substack{\uparrow \\ \text{A, K, Q, or J}}}{\underline{_{16}C_1}} \cdot \underline{_{12}C_1} \cdot \underline{_8C_1} \cdot \underline{_4C_1} = \underline{16}\cdot\underline{12}\cdot\underline{8}\cdot\underline{4} = \boxed{6{,}144 \text{ ways}}$$

17. If the player draws four cards from the deck (same game as above) and the cards drawn are an Ace, a King, a Queen, and a Jack, in that order, the player wins the GRAND SLAM of $100,000,000. How many ways are there to draw a GRAND SLAM in this game?

$$\underset{A}{\underline{_4C_1}} \cdot \underset{K}{\underline{_4C_1}} \cdot \underset{Q}{\underline{_4C_1}} \cdot \underset{J}{\underline{_4C_1}} = \underset{A}{\underline{4}}\cdot\underset{K}{\underline{4}}\cdot\underset{Q}{\underline{4}}\cdot\underset{J}{\underline{4}} = \boxed{256 \text{ ways}}$$

18. How many different ways are there to draw 4 cards from a poker deck, if the order they are drawn in does not matter?

$$_{52}C_4 = \frac{52!}{(52-4)!\,4!} = \boxed{270{,}725 \text{ ways}}$$

$$= \underline{52}\ \underline{51}\ \underline{50}\ \underline{49}$$