

Name Key

Math 102
Practice Test 1- Sets

Show your work whenever appropriate for full credit. (2 pts/problem unless indicated otherwise)

1. Give an example of a finite set that is **not** in roster notation that is well defined.

$$A = \{x : x \text{ is a whole number between } 0 \text{ and } 10\}$$

2. Write out the set $\{x : x \text{ is an integer less than } 4\}$ in roster notation.

$$\{\dots, -3, -2, -1, 0, 1, 2, 3\}$$

3. How many subsets of $\{a, b, c, d, e, f, h, i\}$ are there? Show how you determined it.

$$2^k = 2^8$$

$k = \# \text{ of elements}$

256 subsets

4. Write each of the following in set builder notation.

a. $\{0, 1, 2, 3, 4, 5, 6, 7\}$ $\{x : x \text{ is an integer between } -1 \text{ and } 8\}$

b. $\{\text{January, February, March, April, May}\}$ $\{x : x \text{ is a month of the year between Dec. and Jun.}\}$

$$U' = \emptyset$$

5. Given $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 4, 5, 7\}$ $C = \{7, 8\}$ find:

a. $A \cap B = \{4\}$

b. $B' = \{2, 6, 8, 9\}$

c. $A' \cup B' = \{1, 2, 3, 5, 6, 7, 8, 9\}$

d. $A - B = \{2, 6, 8\}$

$\{1, 3, 5, 7\} \cup \{2, 6, 8, 9\}$

e. $(A \cap C) \cap (A \cup B) = \{8\}$
 $\{8\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

6. List all the subsets of $\{t, i, m\}$.

$\emptyset, \{t\}, \{i\}, \{m\}, \{t, i\}, \{t, m\}, \{i, m\}, \{t, i, m\}$

~~7. Cynthia has 237 minutes remaining on her cell phone this month and wants to use them about equal to call her 11 good friends. Show how to use compatible numbers to determine the number of minutes she can talk to each of her friends.~~

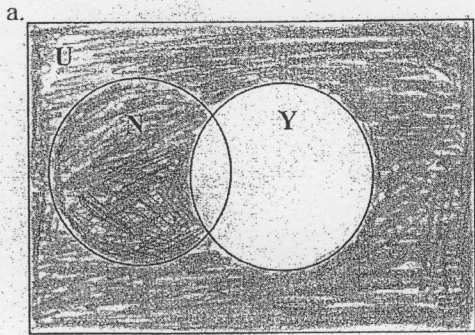
7. Show that the set of whole numbers is infinite.

~~$\frac{240}{10} = \text{about } 24 \text{ minutes}$~~

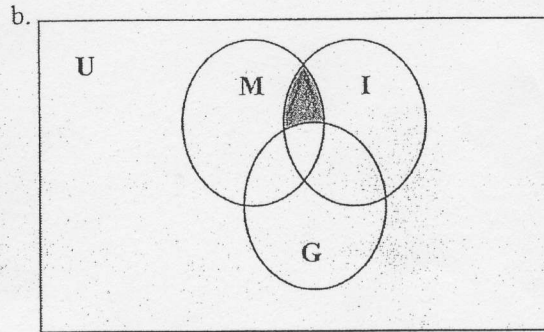
0	1	2	3	4	...	n-1	7.	about 24 minutes
^	^	^	^	^		^		per friend
1	2	3	4	5	...	n		

+ 24

8. What does the shaded region in each figure below represent in set notation?



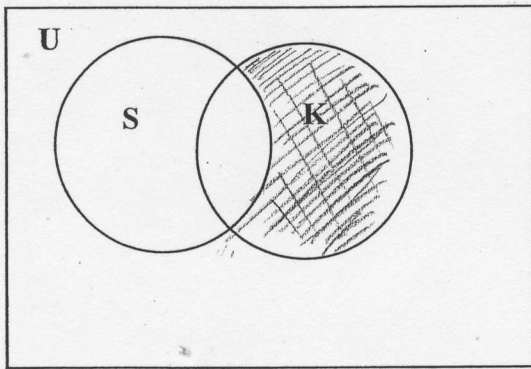
Y' or $U - Y$



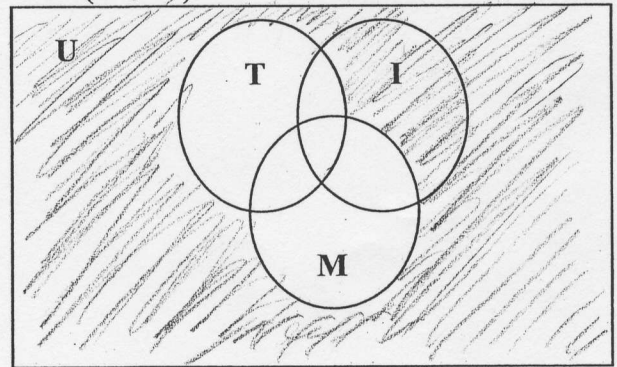
$(M \cap I) - G$

9. Illustrate each of the following by shading the Venn diagrams below.

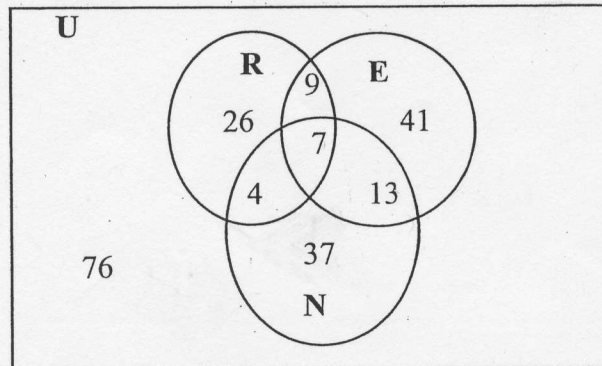
a. $S' \cap K$



b. $(T \cup M)'$



10. Use the diagram below to find the cardinality for each problem.



a. $n(N) = 61$

b. $n[(R \cap E) \cup N] = 70$
 $16 + 4 + 13 + 37$

c. $n(E') = 143$
 $76 + 26 + 4 + 37$

d. $n(E - R) = 54$
 $41 + 13$

e. $n(U) = 213$
 $76 + 26 + 9 + 7 + 4 + 41 + 13 + 37$

f. $n(R \cap E \cap N) = 7$

+20
 20

11. Tell whether each statement is true or false, if false correct the statement to make it true.

a. All sets that are equal are also equivalent.

True $\{1, 2, 3\} = \{1, 2, 3\}$

b. $A' \cap B' = (A \cap B)'$ (Hint: try it using the elements in problem 5 of this test)

False $A' \cap B' = (A \cup B)'$
 $A = \{1, 3, 5, 7, 9\}$ $A' = \{2, 4, 6, 8\}$ $B = \{2, 4, 6, 8, 10\}$ $B' = \{1, 3, 5, 7, 9\}$
 $(A \cap B)' = \{1, 3, 5, 7, 9, 10\}$ $A' \cap B' = \{1, 3, 5, 7, 9\}$
 $(A \cap B)' = \{1, 2, 3, 5, 6, 7, 8\}$

c. If set $F = \{1, 2, 3\}$ and set $G = \{2, 3, 1\}$ then $F \subset G$.

False $F \not\subset G$ $F \subseteq G$

d. For any set $U' = \{ \}$.

True

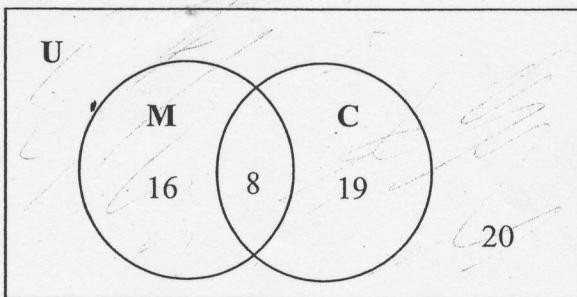
e. The number of proper subsets of a given set is one less than the number of subsets for that same set.

True

f. $n(T \cup L) = n(T) + n(L)$

False $n(T) + n(L)$ counts the intersection twice

12. Use the Venn diagram below to fill in the missing statement (1 pt/problem).



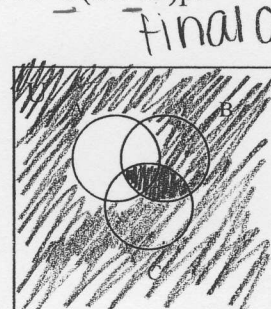
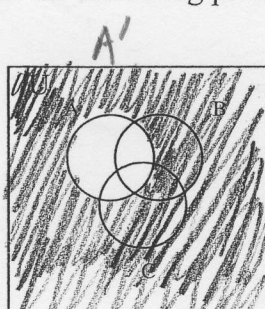
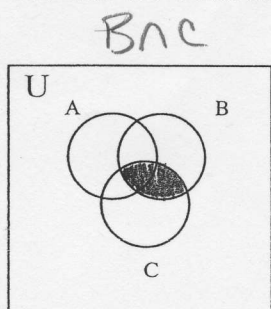
$$M' = 39$$

$$C' = 36$$

a. $n(M' \cup C') = 55$

b. $n(C') = 36$

13. Use the Venn diagram to work the following problem in three steps $A' \cup (B \cap C)$.



$\frac{1}{6}$

14. Fill in the blanks with **one** of the following \in , \notin , \subseteq , $\not\subseteq$, \subset , or $\not\subset$ to make each statement true (1 pt/problem).

a. $\{ \} \underset{\substack{\subseteq \\ \subset}}{\subseteq} \{2, 4\}$

b. Whole $\underset{\substack{\subseteq \\ \subset}}{\subseteq}$ Rational numbers

$\{0, 1, 2, 3\}$ $\{0, 1, 1\frac{1}{2}, 2, \dots\}$

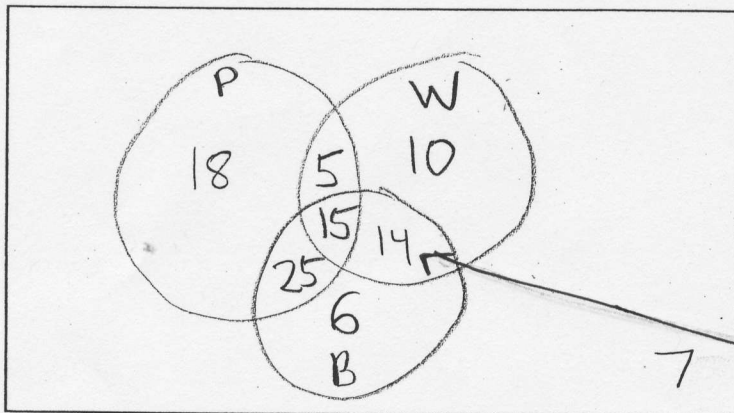
c. $-2 \not\subseteq$ Naturals numbers

d. $-6 \in \{x: x \text{ is a solution to the equation } x^2 = 36\}$

15. There were 100 students in the library who responded to how they completed their research paper.

- ~~18~~ students only used the periodicals \mathcal{P}
- ~~29~~ students used the web and books ~~W & B~~ $29 - 15 = 14$
- ~~15~~ students used books, the web, and periodicals
- ~~40~~ students used books and periodicals
- ~~20~~ used the web and periodicals
- ~~60~~ students used books $\rightarrow 100 - 7 - 6 - 25$
- 7 students did not use the web, nor books, nor the periodicals.

a. Represent this information with a Venn diagram.



\mathcal{P} = periodicals,
 W = web
 B = Books,

$W = 100 - 7 - 6 - 25 - 15 - 14 - 5 - 18$

$29 - 15 = 14$

b. How many students used the web in their research?

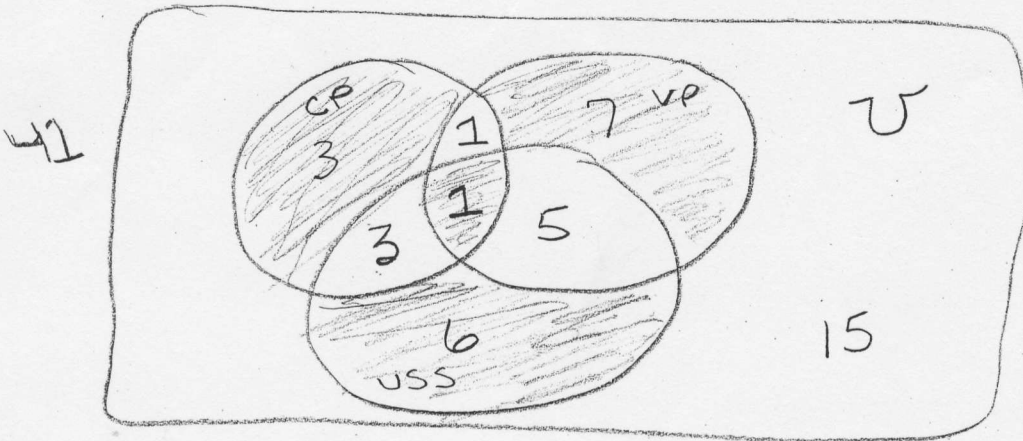
b. 44

c. How many students used books or periodicals?

c. 87

$\frac{710}{10}$

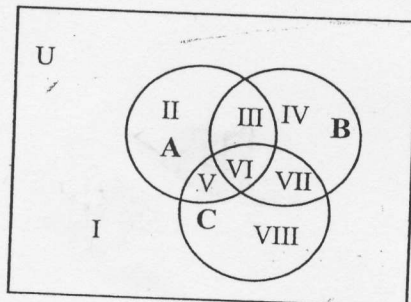
16. Concerning the first 41 presidents of the United States we know the following facts:
Eight held cabinet posts, 14 served as vice-president, 15 served in the U.S. Senate, 2 served in cabinet posts and as vice-president, 4 served in cabinet posts and in the U.S. Senate, 6 served in the U.S. Senate and as vice-president, and 1 served in all three positions (6 pts).



How many presidents served in:

- a. none of these 3 positions? 15
 b. only in the U.S. Senate? 6
 c. at least one of the three position? 26
 d. exactly two positions? 9

17. List the numbered regions that make up the answer to each of the following:



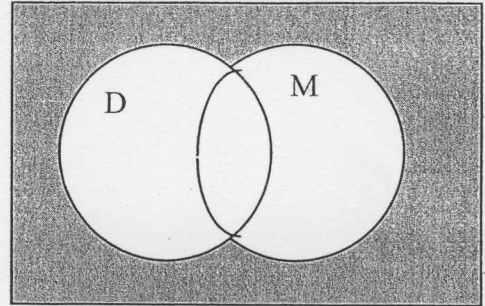
- a. $A - C =$ II, III b. $B' \cap A =$ II, V
 c. $A \cup (B \cap C) =$ II, III, V, VI, VII d. $C \cap (B \cup A)' =$ VIII

$$\frac{21}{14}$$

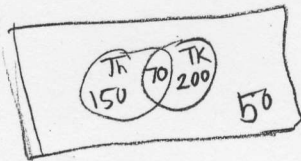
18. Use DeMorgan's Laws and set operations to write the shaded region in two different ways.

$$(D \cup M)' = D' \cap M'$$

$$\overline{U - (D \cup M)}$$



19. A pizza chain is willing to pay \$0.75 to each person interviewed about his or her likes and dislikes of types of pizza crust. Of the people interviewed, 220 liked thin crust, 270 liked thick crust, 70 liked both, and 50 did not like pizza at all. What was the total cost of this survey? (3 pts).



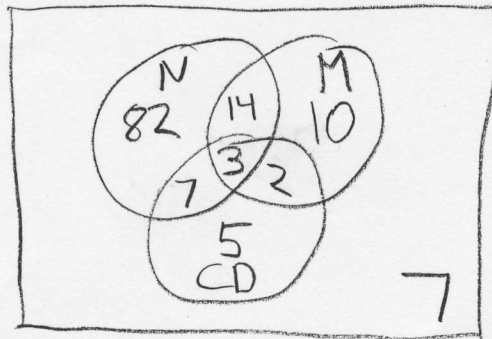
yes

$$50 + 200 + 70 + 150 = 470 \text{ people}$$

$$470 \times 0.75$$

19. \$352.50

20. In a survey of 130 people, the following data were collected: 106 people subscribed to the newspaper, 29 people subscribed to magazines, and 17 people were members of a mail CD club. Seventeen subscribed to both the newspaper and the magazines, 5 people subscribed to magazines and were members of a CD club, and 10 people subscribed to the newspaper and were members of a mail CD club. Three people subscribed to both the newspaper and magazines and were members of a mail CD club. Make and fill in a Venn diagram to illustrate this situation (3 pts).



Bonus Problem: (3 pts) Describe a one-to-one correspondence between the set $\{1/8, 1/27, 1/64, 1/125, \dots\}$ and one of its proper subsets.

~~Solve a problem with 3 strategies~~
~~Visual/picture~~, $1/8 \rightarrow 1/27, 1/64 \rightarrow 1/125 \dots 1/(n+1)^3$
 $1/27 \rightarrow 1/64, 1/125 \rightarrow 1/216 \dots 1/(n+2)^3$