Math 102 Notes for Day 1

Shaking Hands Activity	# People	#Handshakes	Diagrams
a) Greet your classmates with a handshake	2		
b) introduce yourself			
c) keep track of the number of handshakes	3		
d) diagram the # handshakes	4		
⋮			
	n		

Three-Way Principle

______________________ __________________ ________________________

The Nature of Mathematics as an Academic Discipline (Section 1.3)

Arithmetic is only a small part of the field of mathematics.
Mathematics depends on precise definitions and explicitly stated rules of logic.
Mathematical Truth is determined by the definitions and logic being applied.
The mathematical idea of a "set" helps us be precise and define what we are talking about.

So the first two topics we study in this liberal arts survey of mathematics are

- sets
- logic

Notation: the way we write things down in mathematics must reflect this precision. In this course you are required to use "good" notation in all of your written work.

Introduction to Mathematical Sets

Set: a set is a collection of objects.

Element or Elements of a set: any of the objects in the collection. These are also sometimes called members of the set.

- $\in$ is the symbol that means "is an element of".
- $\notin$ is the symbol that means "is not an element of".

"List" or "Roster" Notation: the set is defined by listing all of its elements between set braces { }. Sets are often "named" by assigning capital letters to stand for the whole set.

Ex. A = __________________________________ so ________________ while _______________

Set Builder Notation: the set is described by stating some characteristic that all the elements in the set have in common **and** is not satisfied by any other object. Whether or not an object has this characteristic determines whether the object is in the set.

{____

Universal Set: the set **U** of all elements under consideration in a given discussion or problem is called the universal set.

Empty Set: The empty set has no elements. (If a set has any elements at all, it is not empty). The notation for "empty set" is either { } or ____ . If an equation has no solutions, then the solution set for the equation is the empty set. Another name for the empty set is a **null set**.

Characteristics of Sets

Some characteristics of sets:

- **Order within the set:** the order of the elements in the set does NOT matter.
  Example: {1, 2, 3} is the same set as {2, 3, 1}
- **Distinct elements**: the elements of a set should be distinct, that is, they should each be different from the other. There should be no repeats listed as part of the set.
- **Cardinal number** of a set: The cardinal number of a set tells the number of distinct elements within the set. It is important that we do NOT repeat elements within the set so that when we count them we get the cardinal number of the set. The notation for "the cardinal number of set A" is $n(A)$.
- **Finite set**: A set is finite if its cardinal number s a whole number.
- **Infinite set**: An infinite set is one that is not finite.
  Example: **T** = **{1, 2, 3, 4, . . . }** means the set T contains the numbers 1, 2, 3, 4, and so on continuing in this pattern. The " **. . .** " means "continuing the pattern". When it is the last thing in the list, it means that you continue the pattern forever. Such as set is an **infinite set** because the number of elements in the set is uncountable, they go on forever.
- **Well-defined**: A collection is well-defined if there is not ambiguity as to whether something belongs to the collection or not. So a set is **well-defined** if we are able to tell whether or not any particular object is an element of the set.

Class Practice

1. A = { 1, 2, 3, 4 }. Is {4, 3, 2, 1} the same set? Why or why not?

2. {∅} is not an empty set. Why not?

3. V = {2, 4, 6, 8, **. . .** 26} the " **. . .** " is followed by the ending number. In this case we are told to continue the pattern up through 26 and stop. This is a **finite** set because you can count the number of members in the set. What is $n(V)$? ________

Common Number Sets

SPECIAL SETS OF NUMBERS:

N = Natural numbers = {1, 2, 3, 4, . . . } (positive counting numbers)

W **=** Whole numbers = {0, 1, 2, 3, . . . } (positive counting numbers and zero)

J = Integers = { . . . –2, –1, 0, 1, 2, 3 . . . } (positive and negative counting numbers and zero)

Q = Rational Numbers **=** $\{\frac{m}{n} \mid m \text{ and } n \text{ are} \in J \text{ and } n \neq 0\}$

(numbers that can be written as fractions of integer numbers)

R = Real Numbers: all the numbers that express distances from the origin on a number line.

Real Numbers include:

- **all integers** (positive counting numbers, negative counting numbers, and zero)
- **all fractions and decimals** (non-integer, rational numbers)
- **irrational numbers such as** $\sqrt{3}$, $\sqrt{5}$, π, $\frac{\pi}{2}$**,** e which cannot be written as fractions or as decimals that end or repeat. Irrational numbers are decimals that never end and never repeat. This is why we frequently use a symbol, such as π or e, to represent the number, because anything else we would write would just be an approximation..

Applying the definitions: Put a √ to indicate which set each number belongs to.
NOTE: a number may belong to more than one set.

Number	***N***	***W***	***J***	***Q***	***R***
7					
0					
–4					
2.4					
$\frac{2}{3}$					
2π					
.121221222…					
.2323232323…					
$\sqrt{2}$					

Terminology Examples

1. List the set of integers between -5 and 2, not inclusive. ______________________________

2. List the set of multiples of 5 greater than 10. ____

3. List the set of natural numbers between -5 and 2, inclusive. ___________________________

4. Express {1, 2, 3, 4, 6, 12} as a set using set builder notation.

   ____

5. Express $\{x : x \text{ is an integer and } x^2 = 100\}$ in roster notation.

   ____

6. Express $\{x : x \text{ is a president of MSUM between 1980 - 2009}\}$ using the listing method.

   ____

7. Express $\{x : x \text{ is a natural number and } x^2 = 100\}$ in roster notation. __________________

8. List the set of integers which when squared equal 11. ________________________________

9. List the whole number multiples of 2. ____

10. For $C = \{a, 6, *, y, 9\}$ $n(C) =$ ________________